Lecture 1:
Financial Frictions Before the Flood

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Introduction

- Keynesian IS-LM models came to be used for policy analysis and forecasting during the 1960s.
- However, Keynesian theory was built initially to explain two Great Depression observations. These are
  1. short nominal rates were near zero for much of the 1930s,
  2. while monetary policy appeared to be ineffective.
- The answer is the liquidity trap $\Longrightarrow$ monetary policy fails because it "pushs on a string" at the zero lower bound.
- This is a symptom of multiple equilibria.
  1. Multiple equilibria is the key idea of Keynes' general theory.
  2. $\Longrightarrow$ Insufficient aggregate demand is the problem because it fails to meet aggregate supply.
  3. A Keynesian policy response is a fiscal expansion to create the missing aggregate demand.

Critiques of the Liquidity Trap

- Implicit in many Keynesian models, traditional and new, is consumption, investment, and labor demand choices are independent of financial decisions (*i.e.*, portfolio allocation).
- Minus this assumption, Keynesian predictions about the liquidity trap and aggregate demand management are altered if not negated.
- This critique of Keynesian theory is associated with Brainard and Tobin (Yale) and Brunner and Meltzer (Carngie-Mellon/Rochester).
- Starting in the 1960s, they create models in which households and firms face incomplete financial markets to study the impact on the monetary transmission mechanism.
    1. There are several assets, which are not perfect substitutes
    2. Monetary non-neutralities exist because private agents lack complete financial markets in which to hedge a change in central bank policy.
    3. There are several independent interest rates on which to compute equilibria $\Longrightarrow$ multiple interest rate margins on which to operate monetary policy.

Tobin (AER–PnP, 1961): Introduction

- Tobin introduces ideas that languish in macro for 30 years.
- The ideas are financial frictions, incomplete markets, and risk and uncertainty may matter for aggregate fluctuations.
- The opposite is typical of Keynesian models, traditional and new.
  1. $\Longrightarrow$ Assets are perfect substitutes except for fiat currency.
  2. $\Longrightarrow$ Changes in the stock or intertemporal price of a central bank's liability can alter real allocations.
- If assets are not perfect substitutes, many interest rates exist independently in general equilibrium.
  1. These interest rates are functions of the primitives of the economy.
  2. $\Longrightarrow$ Changes in the stock or intertemporal price of many private (nearly riskless) assets/liabilities can also alter real allocations.

Tobin (AER–PnP, 1961): The monetary transmission mechanism

- Tobin argues a complete theory of the monetary transmission mechanism involves
  1. short- and long-term private assets and government debt,
  2. short- and long-term private and government interest rates,
  3. and private sector and government portfolio decisions, which contribute to equilibrium rates.
- Preferences, technology, and market structure drive decisions by private agents and governments
  1. about the flows and stocks of their assets and liabilities.
  2. These decisions determine private and public balance sheets,
  3. but the decisions are over flows that become stored in stocks.
- Technology is about production of goods and services while market structure describes the form of market incompleteness.
  1. Production of assets is also part of technology $\Longrightarrow$ transaction, intermediation, and monitoring costs in financial markets.
  2. These costs are frictions shaped by which markets are missing.

Tobin (AER–PnP, 1961): Settling Accounts

- Monetary theory is more than determining the aggregate price level.
- Since Fisher (1911), economists have known that private liabilities or debt (*i.e.*, inside money), can influence the aggregate price level by means of the quantity equation.
- Tobin aims to move beyond the quantity theory to incorporate
  1. the real effects of changes in the equilibrium values of the stocks of all government liabilities.
  2. $\Longrightarrow$ Equivalent to changes in the (net) claims on the government (*i.e.*, liabilities of the treasury and central bank).
  3. These liabilities are more than changes in the supply of fiat currency vailable to settle most payments.
  4. Does the payments system accept interest bearing government debt to settle accounts?
  5. $\Longrightarrow$ Does government debt has net worth or real value?
  6. Some private debts are able to extinguish claims on unpaid accounts (*i.e.*, bankers' acceptances and bills of exchange).
  7. Are private liabilities and government debt money?

Tobin (AER–PnP, 1961): The Monetary Operating Mechanism

- Modern macro models often treat the monetary authority's interest rate rule as exogenous.
- Or are the "instruments of monetary control" the true exogenous elements of monetary policy?
- The instruments of monetary control include
  1. the central bank's discount rate, reserve requirements, the rate of interest on (excess) reserves (if being paid), the target policy rate,
  2. (but not the policy rate "set in the market"), and the supply of the central bank's liabilities.
- What are the exogenous liabilities of central banks?
  1. Fiat currency or bank reserves? Or something else?
  2. Bank reserves are liabilities of central banks, but are these liabilities relevant for the mapping from the monetary operating mechanism to the monetary transmission mechanism?
  3. Is this true only if private banks define the universe of FIs and at the margin these firms fund long maturity assets on their balance sheets with reserves?

Tobin (AER–PnP, 1961): Monetary Policy and Wicksellian Rates, I

- Tobin wants to understand which economic object(s) a central bank should target, given the goal of monetary policy is aggregate price level stability.
- New Keynesian (NK) models equate (the inverse of) the policy rate with the household's stochastic discount factor ($SDF_{t,t+1}$) multiplied by the real return to currency, $R_{p,t}^{-1} = \mathbf{E}_t \left\{ SDF_{t,t+1} \left(1 + \pi_{t+1}\right)^{-1} \right\}$.
- The SDF is the natural rate of interest in NK models.
  1. Assuming no drift in inflation, the canonical NK model predicts
  2. a central bank targets $R_{p,t}^{-1} = \mathbf{E}_t SDF_{t,t+1}$ to achieve $\mathbf{E}_t \pi_{t+1} = 0$.
  3. $\Longrightarrow$ The natural rate prescription that achieves optimal policy.
  4. But a natural rate model has only one independent interest rate $\Longrightarrow$ the intermediate target for an inflation targeting central bank.
- Tobin's analysis suggests flaws in the NK model of monetary policy.
  1. Whether financial frictions are produced by incomplete markets, imperfect asset substitution, or transactions costs, there exist several independent interest rates in equilibrium,
  2. In this case, which interest rate does the central bank target?

Tobin (AER–PnP, 1961): Monetary Policy and Wicksellian Rates, II

- Example: Wicksell (1898) identified the natural rate with the return (*i.e.*, the marginal product, $\text{MP}_{K,t}$) on capital and the rate on bank loans, $R_{\ell,t}$, with the market interest rate.
  1. The "gap" between $\text{MP}_{K,t}$ and $R_{\ell,t}/(1+\pi_t)$ is driven by banks issuing loans (*i.e.*, inside money creation).
  2. Borrowers take loans from banks if $\text{MP}_{K,t} \geq R_{\ell,t}/(1+\pi_t)$ $\Longrightarrow$ inside money creation leads to growth in the price of capital and perhaps other factors that are inputs into production.
  3. Opposite occurs when $\text{MP}_{K,t} < R_{\ell,t}/(1+\pi_t)$, which can generate a deflation in prices of assets and other factors of production.
- This Wicksellian natural rate dynamic is not necessarily equivalent to inflation, disinflation, or deflation in the aggregate price level.

Tobin (AER–PnP, 1961): Monetary Policy and Wicksellian Rates, III

- Tobin's monetary business cycle theory is about the response of the aggregate price level to the $\text{MP}_{K,t}$ net of the (real) rate at which investors are willing to hold capital, $r_{K,t}$.
  1. This spread depends, in part, on how investors fund their portfolios.
  2. In the U.S. in 1961, bank loans were the main source of credit for investors to purchase long maturity assets.
  3. $\Longrightarrow$ The Fed's control of excess or unborrowed reserves was a key source of its ability to affect interest rates.

- The spread $\text{MP}_{K,t} - r_{K,t}$ is Tobin's Wicksellian gap and the central bank's intermediate target, but compare term spreads, Figure Term Spreads, and short and long spreads, Figure Short & Long Spreads, of private and U.S. Treasury rates from $1920Q1$ to $2015Q4$ $\Longrightarrow$ which spread should a central bank target?

- However, Tobin arrives at this point only after arguing
  1. there are multiple interest rates that matter for real allocations in equilibrium.
  2. $\Longrightarrow$ Market imperfections cause assets to be imperfect substitutes, government debt may have real net worth, interest bearing private and government debt may circulate as money to settle payments,
  3. and monetary policy should operate on the exogenous instruments available to a central bank.

Brainard and Tobin (AER–PnP, 1963)

- Brainard and Tobin start from Tobin's model and add FIs.
  1. They study the monetary transmission mechanism by focusing
  2. on the flows into and out of the assets and liabilities on the balance sheets of FIs.

- Instead of having individual savers lend to borrowers, FIs take on the task of transforming savings into credit because this
  1. pools or diversifies the risk of these capital projects,
  2. minimizes the costs to administer and monitor loans.

- The monetary transmission mechanism relies on incomplete financial markets to generate monetary non-neutralities.
  1. A central bank alters $\text{MP}_{K,t} - r_{K,t}$ by trading its liability for assets on FI balance sheets $\Longrightarrow$ open market operations (OMOs).
  2. Tobin and Brainard contend this mechanism is effective whether or not FIs are regulated $\Longrightarrow$ FIs with balance sheets constrained by government dictums.

- These lectures regard the last point an open research question.

Brunner and Meltzer (AER–PnP, 1988)

- Brunner and Meltzer (BM) argue there are fundamental problems with the Keynesian liquidity trap.
  1. Keynesian models lack a rich financial market structure.
  2. Credit markets are fused to money markets and/or
  3. financial securities and real assets perfect substitutes by assumption.
- The Banking Act of 1933 separated commercial banks from securities firms.
  1. Commercial banks take deposits and securities firms that underwrite new stock and bond issues.
  2. $\Longrightarrow$ A regulatory friction separated U.S. credit and money markets.
  3. Sargent (2011) discusses these issues in the context of U.S. monetary history and several macro models; also see his Phillips Lecture: Uncertainty and Ambiguity in American Fiscal and Monetary Policies at the LSE in February 2010, which is the source of his paper.
- The money markets, which are sometimes called the interbank markets, are where financial firms borrow from and lend to each other.
- Private agents borrow from financial firms in the credit markets.

Money and Credit

- Given a credit market, financial intermediaries (FIs) have a role to play in allocating household wealth to productive uses.
- BM study the effect on the monetary transmission mechanism of shocks to the relative price of money for
  1. credit,
  2. real assets, and
  3. the service flows provided by real assets.
- In response to a monetary shock, investors could alter their portfolios along the money, financial security, or real asset margins.
- The direction and magnitude of the portfolio shifts depend on the relative interest elasticities of money demand and credit demand.

The BM Monetary Business Cycle

- Incorporating a credit market into an IS-LM model affects its properties and business cycle predictions.
- The IS-LM model will not necessarily produce
  1. interest elasticities with the same sign and
  2. there is no liquidity trap.
- Since the signs of the interest elasticities can differ, the response of output to a credit market shock is not similar to the impact of a supply shock.
- BM argue there is no liquidity trap because a short nominal rate near zero
  1. does not preclude other intertemporal relative price movements generating money market fluctuations that feed into output changes.
  2. Figure Rates depicts episodes from $1920Q1$ to $2015Q4$ during which only short rates are near zero.
- Thus, an interest rate rule will not necessarily yield a monetary business cycle independent of money demand/supply shocks.

BM's Policy Analysis

- Investors react to relative price movements across money, credit, and asset markets.
- Thus, real and nominal shocks drive movements in the credit market that produce business cycle fluctuations.
- BM interpret this prediction of their IS-LM model as showing the need to focus monetary policy on credit aggregates (*i.e.*, M2).
- A related issue is whether disturbances in the credit market are an independent source of exogenous impulse into the real economy.

FI Balance Sheets and Monetary Transmission

- A credit market connects the balance sheets of FIs to the liability issued by a central bank, which is outside money.
- FI balance sheets become a potential monetary transmission mechanism when the demand for credit is more elastic than the demand for money.
- In this case, the credit market response to a given shock dominates the response of the money market.
- FIs can react by changing the liability or asset side of their balance sheets.
  1. Changes in credit supply (demand) operate on the liability (asset) side of a FI's balance sheet.
  2. The liability of FIs is often called inside money.
  3. At the margin, when FIs alter the supply of inside money, liquidity in the credit market changes.
  4. What is the relative price of inside money to the outside money?
  5. Do FIs adjust on the liability side of their balance sheets because of liquidity or collateral constraints? Or another reason?

Financial Crises and Monetary Shocks

- Suppose there is a negative shock of extraordinary magnitude that raises the demand for cash and/or lowers the supply of credit.

- FI reduce their activities, which reduces liquidity in credit markets.
  1. The fall in liquidity can produce bank failures.
  2. A central bank using its lender of last resort (LLR) authority can replace the missing liquidity to avert bank failures.

- This still begs the question of the source(s) and cause(s) of the shock that initiated the financial crisis.

- BM argue that financial crises are endogenous events that originate in poor decisions by central banks (and Treasuries).
  1. The credit market propagates monetary shocks into the real side of the economy.
  2. Thus, monetary shocks are the source and cause of financial crises.

- This is not a consensus view.

The Liquidity and Expected Inflation Effects

- Putting aside the issue of financial crises, BM take the position that monetary policy has persistent real effects on output because of credit market activity.
- If the credit market is the source of monetary non-neutralities over the business cycle, the "liquidity effect" explains the monetary transmission mechanism.
- An alternative to the liquidity effect is the expected inflation effect.
- Most monetary DSGE models are driven by the expected inflation effect. (Hint: Shocks to expected inflation drive monetary non-neutralities in new Keynesian models.)
- Lucas (JET, 1990) and Fuerst (JME, 1992) seek to construct a monetary DSGE model that predicts a dominate liquidity effect.

Liquidity, Expected Inflation, and Interest Rates

- Consider the textbook Fisher relation $R_t = r_t + \mathbf{E}_t \pi_{t+1}$, where R_t, r_t, and $\mathbf{E}_t \pi_{t+1}$ are the nominal rate, the real rate, and expected inflation.
- The expected inflation and liquidity effects can appear in the Fisher equation in several ways.
- One example is $R_t = r_t + \mathbf{E}_t \pi_{t+1} + \varpi \mathbf{E}_t g_{\pi,t+1}$, where $\mathbf{E}_t g_{\pi,t+1}$ is expected inflation growth.
- Expected inflation dominates the liquidity effect if persistent increases in $\mathbf{E}_t g_{\pi,t+1}$ raise R_t $(-1 < \varpi)$.
- The liquidity effect can also be present when $\varpi < -1$ or if an increase in a real variable raises R_t as, for example, in

$$R_t = r_t + \mathbf{E}_t \pi_{t+1} + \mathbf{E}_t g_{y,t+1},$$

where $g_{y,t+1}$ is output growth.

But What's in a Name?

- Financial and macro economists have called theories that invoke the impact of the credit market on real activity by several different names.
- These are the liquidity effect, loanable funds theory, the credit channel, and the balance sheet effect.
- There are differences, but these range from minor to subtle.
- The Lucas-Fuerst, BGG, and Kiyotaki–Moore models are useful examples that explain the different approaches to integrating the credit market into macro theory.

The Lucas-Fuerst Notion of Liquidity and Loanable Funds

- Fuerst (JME, 1992) argues that the liquidity effect is separate from the loanable funds effect.

- This version of the liquidity effect, adopted from Friedman and Schwartz, starts with expansionary monetary policy leaving the economy with more cash.
    1. Whoever receives the additional cash uses it to buy either interest bearing securities or durable goods.
    2. In general equilibrium, interest rates and asset prices change.

- Loanable funds is only a label to remind us that FIs receive cash from the monetary authority, not households or firms.

- The Lucas-Fuerst (LF) model assumes that FI loan the excess cash to firms to pay for productivity activity.

- These loans appear as assets on the FIs balance sheet.

The LF Model

- The economy consists of a representative firm, FI, household, and a monetary authority.
- All markets are perfectly competitive.
- All agents form rational expectations (RE).

The Firm of the LF Model

- The firm mixes capital and labor to produce output.
  1. Capital is owned by the firm and is accumulated from retained profits.
  2. Workers are hired in a competitive spot labor market.
  3. The firm finances its nominal wage bill by borrowing from the FI.
  4. The rest of firm profits are paid as dividends to the household.

The FI of the LF Model

- The FI accepts household deposits at the beginning of date t, which back the *intra*–period loans made to the firm during that period.
  1. There are neither financial frictions nor is there a CRS intermediation technology that transforms deposits into loans (*i.e.* the FI has no need for capital or to hire workers to engage in intermediation).
  2. The monetary authority alters the resources available for loans by injecting or withdrawing cash from the FI's balance sheet.
  3. The FI turns a dollar of deposits into a dollar of loans net of the monetary authority's cash injection or withdrawal.
  4. At the end of date t, loans and deposits are paid off with interest.
  5. FI profits are paid as dividends to the household.

The Limited Participation Assumption of LF

- The household sends a worker to the labor market and a saver to the FI at the start of date t.
  1. The saver deposits the household's saving with the FI in return for those dollars plus interest at the end of date t.
  2. The worker sells labor services to the firm at the equilibrium nominal wage.
- The household is limited in its actions when participating in the labor and "money" market in the FL model.
- Thus, the limited participation moniker of the FL model.

The Lucas-Fuerst Financial Friction

- LF impose a timing friction on the economy.
- The household deposits cash with the FI at the beginning of date t, which is prior to that period's monetary policy action.
  1. The FI offers a loan to the firm subsequent to the monetary authority's date t action.
  2. A change in monetary accommodation moves the FI along its loanable fund schedule.
- Thus, the household faces an infinite cost of adjusting their investment decisions during date t.

The Limited Participation Model: The Household's Problem

$$\mathcal{J}\left(K_t,\, \frac{M_t}{P_t},\, A_t,\, \mu_t\right) \;=\; \mathbf{Max}_{\left(C_t, N_t, d_t, M_{t+1}\right)}\Bigg[V\Big(C_t,\, 1 - N_t\Big) \\ \qquad + \;\; \beta\mathbf{E}_t\left\{\mathcal{J}\left(K_{t+1},\, \frac{M_{t+1}}{P_{t+1}},\, A_{t+1},\, \mu_{t+1}\right)\right\}\Bigg],$$

subject to the budget constraint

$$\frac{W_t}{P_t}N_t + \frac{D_{Y,t}}{P_t} + \frac{D_{FI,t}}{P_t} + (1 + R_{d,t})\frac{d_t}{P_t} + \frac{M_t}{P_t} = C_t + \frac{d_t}{P_t} + \frac{M_{t+1}}{P_t},$$

and the cash-in-advance (CIA) constraint

$$C_t \;\leq\; \frac{W_t}{P_t}N_t + \frac{M_t}{P_t} - \frac{d_t}{P_t}.$$

given K_0, M_0, P_0, A_0, and μ_0, where K_t, M_t, P_t, A_t, μ_t, C_t, N_t, W_t, $D_{Y,t}$, $D_{FI,t}$, $R_{d,t}$, and d_t denote the capital stock at end of date t, the stock of cash at the end of date t, the price level, TFP, money growth, household consumption, labor supply, the nominal wage, dividends the firm pays to the household, dividends the FI pays to the household, the nominal return on deposits, and deposits the household leaves at the FI, respectively.

The Limited Participation Model: The Household's FONC

$$\begin{aligned}
C_t: &\quad V_{C,t} - \lambda_{1,t} - \lambda_{2,t} = 0, \\
N_t: &\quad -V_{N,t} + \left(\lambda_{1,t} + \lambda_{2,t}\right)\frac{W_t}{P_t} = 0, \\
d_t: &\quad \mathbf{E}_{t-1}\left\{(1 + R_{d,t})\lambda_{1,t} - \lambda_{1,t} - \lambda_{2,t}\right\} = 0, \\
M_{t+1}: &\quad -\frac{\lambda_{1,t}}{P_t} + \beta\mathbf{E}_t\left\{\frac{\partial \mathcal{J}_{t+1}}{\partial M_{t+1}}\right\} = 0,
\end{aligned}$$

where $\lambda_{1,t}$ is the Lagrange multiplier attached to the household's budget constraint and $\lambda_{2,t}$ is the Lagrange multiplier attached to the household's CIA.

The Limited Participation Model: Household Optimality

Trade-offs

$$C_t\text{–}N_t: \quad \mathbf{E}_t\left\{\frac{P_t}{P_{t+1}}V_{C,t+1}\right\}\frac{W_t}{P_t} = V_{N,t},$$

$$C_t\text{–}M_{t+1}: \quad \mathbf{E}_{t-1}\left\{\frac{V_{C,t}}{P_t} - \beta(1+R_{d,t})\frac{V_{C,t+1}}{P_{t+1}}\right\} = 0,$$

where $\frac{\partial \mathcal{J}_t}{\partial M_t} = \frac{V_{C,t}}{P_t}$, which assumes the Benveniste and Scheinkman (1979, Econometrica) conditions hold.

- The CIA prevents the household from trading date t leisure for the consumption, and hence additional utility, that is received when supplying an extra unit of labor during date t.
- The household decides whether to deposit some of M_t with the FI or send it to the goods market to purchase more consumption prior to observing the monetary shock of date t.

The Limited Participation Model: The Firm's Problem

$$\textbf{Max}_{\left(H_t,\, l_t,\, K_{t+1}\right)} \mathbf{E}_t \left\{ \sum_{j=0}^{\infty} \beta^j \frac{V_{C,t+1+j}}{P_{t+1+j}} D_{Y,t+j} \right\}$$

subject to the profit constraint

$$D_{Y,t} \;=\; P_t\Big(Y_t \;-\; \big[K_{t+1} - (1-\delta)K_t\big]\Big) \;+\; l_t \;-\; W_t H_t \;-\; (1 + R_{l,t})\, l_t,$$

and the intra–period loan (or credit)-in-advance (LIA) constraint

$$W_t H_t \;\leq\; l_t,$$

given K_0, where output, Y_t, is produced with the constant returns to scale (CRS) production function $\mathcal{F}\left(K_t,\, A_t H_t\right)$, and H_t, $R_{l,t}$, and l_t denote the firm's labor demand, the nominal rate the FI charges the firm on loans, and the (nominal) loan the firm receives from the FI, respectively.

The Limited Participation Model: The Firm's FONC

$$
\begin{aligned}
H_t: \quad & \Lambda_t W_t &=& \left[\mathcal{F}_{H,t} - \frac{W_t}{P_t}\right]\mathbf{E}_t \frac{P_t}{P_{t+1}} V_{C,t+1}, \\
l_t: \quad & \Lambda_t &=& \frac{R_{l,t}}{P_t}\mathbf{E}_t \frac{P_t}{P_{t+1}} V_{C,t+1}, \\
K_{t+1}: \quad & \mathbf{E}_t \frac{P_t}{P_{t+1}} V_{C,t+1} &=& \beta \mathbf{E}_t \left\{ \frac{P_{t+1}}{P_{t+2}} V_{C,t+2}\left[\mathcal{F}_{K,t+1} + (1-\delta)\right]\right\},
\end{aligned}
$$

where Λ_t is the Lagrange multiplier attached to the firm's LIA.

The Limited Participation Model: Firm Optimality

Trade-offs

$$D_{Y,t}\text{-}N_t: \quad \Big(1 + R_{l,t}\Big)W_t \quad = \quad P_t\mathcal{F}_{H,t},$$

$$D_{Y,t}\text{-}K_{t+1}: \quad \mathbf{E}_t\,\frac{P_t}{P_{t+1}}V_{C,t+1} \quad = \quad \beta\mathbf{E}_t\left\{\frac{P_{t+1}}{P_{t+2}}V_{C,t+2}\Big[\mathcal{F}_{K,t+1} \;+\; (1-\delta)\Big]\right\}.$$

- The firm does not set its labor demand (*i.e.*, the real wage it offers in the spot labor market) equal to the marginal product of labor.
- Instead, nominal marginal revenue w/r/t labor input equals the nominal wage bill plus the cost of the loan the firm needs to finance that factor input.
- Standard monetary economy's consumption-capital Euler equation modified for household's CIA.

The Limited Participation Model: The FI Problem

$$\mathbf{Max}_{\left(l_t,\, d_t\right)} \mathbf{E}_t \left\{ \sum_{j=0}^{\infty} \beta^j \frac{V_{C,t+1+j}}{P_{t+1+j}} D_{FI,t+j} \right\}$$

subject to the profit constraint

$$D_{FI,t} = (1 + R_{l,t})l_t - (1 + R_{d,t})d_t + X_t - (l_t - d_t),$$

and the balance sheet adding up condition

$$d_t + X_t \leq l_t,$$

where X_t $(= M_{t+1} - M_t)$, is the cash the monetary authority adds or subtracts from the economy during each period by altering the stock of the household's liabilities.

The Limited Participation Model: FI Optimality

- The FI is nothing more than a balance sheet.
- The balance sheet constraint, $d_t + X_t \leq l_t$, restricts the FI's liabilities to less than or equal to the FI's assets.
- The FI has no technology to produce loans out of deposits.
- The FI faces no frictions when its liabilities back loans.
- The deposit and loan markets are perfectly competitive.
- Since the FI faces the zero profit condition

$$R_{d,t} d_t = R_{l,t}[l_t - X_t],$$

a RE equilibrium requires that $R_{d,t} = R_{l,t} \equiv R_t$.

The Limited Participation Model's Equilibrium and Optimality

Equilibrium

$$\begin{aligned}
\text{Labor Market}: \quad H_t &= N_t \\
\text{Goods Market}: \quad Y_t &= C_t + K_{t+1} - (1-\delta)K_t. \\
\text{Credit Market}: \quad d_t &= l_t - X_t. \\
\text{Money Market}: \quad M_t &= P_t C_t - X_t.
\end{aligned}$$

Trade-offs

$$C_t\text{-}N_t: \quad \Big(1 + R_t\Big) V_{N,t} \left[\mathbf{E}_t \frac{P_t}{P_{t+1}} V_{C,t+1} \right]^{-1} = \mathcal{F}_{H,t},$$

$$C_t\text{-}M_{t+1}: \quad \mathbf{E}_{t-1} \left\{ \frac{V_{C,t}}{P_t} \left[1 - \beta \frac{1 + R_t}{1 + \pi_{t+1}} \frac{V_{C,t+1}}{V_{C,t}} \right] \right\} = 0.$$

The Limited Participation Model's Equilibrium and Optimality

- The economy's trade-off between leisure and consumption involves a credit market variable, which is the equilibrium rate of return, R_t.
- The intertemporal optimality condition suggests R_t responds to changes in inflation and the marginal utility of consumption between dates t and $t+1$.
- Since $V_{C,t}$ and P_t cannot be zero in a RE equilibrium, the money-credit market arbitrage condition is

$$\mathbf{E}_{t-1}\left\{\frac{1+R_t}{1+\pi_{t+1}}\,\frac{\beta V_{C,t+1}}{V_{C,t}}\right\} = 1.$$

- The arbitrage condition restricts the Fisher equation of the LF model $\Longrightarrow$ the expected nominal rate is a function of expectations of inflation and the real rate, which is the stochastic discount factor (SDF) = $\beta V_{C,t+1}/V_{C,t}$.

The Limited Participation Model's Fisher Equation

- The LF model's Fisher equation suggests the Euler error

$$\xi_{t+1|t-1} = \frac{1+R_t}{1+\pi_{t+1}}\frac{\beta V_{C,t+1}}{V_{C,t}} - \mathbf{E}_{t-1}\left\{\frac{1+R_t}{1+\pi_{t+1}}\frac{\beta V_{C,t+1}}{V_{C,t}}\right\}.$$

- Substitute the Euler error into the Fisher equation to find

$$\frac{1+R_t}{1+\pi_{t+1}}\frac{\beta V_{C,t+1}}{V_{C,t}} = \xi_{t+1|t-1},$$

  or subsequent to passing the natural log operator through

$$R_t \approx \pi_{t+1} - \ln\beta - \left(\ln V_{C,t+1} - \ln V_{C,t}\right) + \ln\xi_{t+1|t-1}.$$

- Whether movements in π or the SDF generate larger changes in R_t is an empirical question.

Cook (JME, 1999)

- The idea of production externalities is adapted to give the FI a technology for transforming deposits into loans.
- The intermediation technology lowers the cost of evaluating and monitoring loans that is a function of aggregate activity.
  1. Since intermediation costs depend on aggregate activity, R_l has a "social return" component along with a private return.
  2. When the social return varies more than the private return, the loan-deposit spread is driven by movements in intermediation costs.
- Cook calls this the "intermediation cost channel" and claims it produces a liquidity effect.

Social Returns, the Cost Channel, and the Liquidity Effect

- Assume that during each period there is a probability, $q \in (0, \;\; 1)$, that the firm will default on its loan.
- The FI's evaluation and monitoring technology is the cost function $\phi(\cdot)$, where $\phi'(\cdot) < 0$ to reflect intermediation costs are a decreasing function of aggregate activity.
- The cost of intermediation, $\phi(\cdot)$, is bounded above by $R_{l,t} - R_{d,t} + qR_{d,t}$ at each date t.
- That is, $\phi(\cdot)$ cannot be high when aggregate activity is low.
- Although there is a "social return" element to the gap between R_l and R_d, $\phi(\cdot)$ is covering the cost of loan losses for the FI.

The FI's Intermediation Cost Technology

- Cook assumes that a dynamic cost of adjustment process generates the stock variable, Z_t, from which flows the social return on aggregate activity.

- The cost of adjustment process is a convex combination of lagged Z_t and the current flow of aggregate employment, H_t,

$$Z_t = (1 - \varrho)Z_{t-1} + \varrho H_{t-1}, \ \ \varrho \in (0, 1).$$

- The FI needs to purchase output from the firm to operate the intermediation cost technology, $C_{FI,t} = \phi(Z_t)\frac{l_t}{P_t}$.

Revising the FI's Dividend and Zero Profit Conditions

- The balance sheet of the FI is unchanged.
- The dividend constraint of the FI becomes

$$D_{FI,t} \;=\; (1+R_{l,t})l_t \;-\; (1+R_{d,t})d_t \;+\; X_t \;-\; (l_t \;-\; d_t) \;-\; P_tC_{FI,t}.$$

- By implication, the economy's aggregate resource constraint accounts for the FI's consumption of the single (physical) good of the economy, $Y_t = C_t + C_{FI,t} + K_{t+1} - (1-\delta)K_t$.
- The zero profit condition is, $R_{l,t} = R_{d,t} + \phi(Z_t)l_t$, which follows from the equilibrium
  1. $D_{FI,t} = (1+R_{d,t})X_t$,
  2. $D_{FI,t} = (1+R_{l,t})l_t - (1+R_{d,t})d_t - \phi(Z_t)l_t$,
  3. and the balance sheet constraint, $l_t = d_t + X_t$.

Intermediation Costs and Monetary Propagation

- The wedge between the $R_{l,t}$ and $R_{d,t}$ is time-varying and a function of the aggregate state of the economy, $\phi(Z_t)$.
- Consider the experiment of a monetary expansion that
  1. reduces $R_{l,t}$,
  2. raising aggregate activity and therefore H_t, which
  3. lowers future intermediation costs pushing H_t still higher.
- This is a virtuous cycle because decreasing intermediation costs causes R_l to fall which is fuel for the firm to borrow more to finance an increasing demand for labor.
- Does this depend on a state dependent probability of loan default q?

Gordon & Leeper: The Price Level, Financial Services, and Government Policies

- G&L study monetary equilibria in a DSGE model that has structure similar and dissimilar to the LF model.
    1. Similar: the household has to carry cash from date $t+1$ to date t to purchase goods of the later date, a firm uses physical capital to produce goods, a firm providing financial services, and monetary policy.
    2. Dissimilar: financial services are an alternative to settle payments for goods rather than intermediation and the government conducts monetary policy and fiscal policy, which proportional taxation of goods output and issuing nominal interest bearing debt.

- The motivation for the dissimilar features are to analyze the interactions of monetary and fiscal policies.

- G&L construct and specify their DSGE model to create analytic (*i.e.*, closed form) equilibrium decision rules.

- The equilibrium decision rules are employed to study the impact of monetary policy and fiscal policy interactions on
    1. the determination of the aggregate price level and nominal interest rate,
    2. portfolio choices of physical capital, fiat currency (*i.e.*, household money demand), and nominal interest bearing government debt, and
    3. quantity and price responses to changes in monetary and fiscal policy.

G&L: Model Structure

- The G&L construct a two sector DSGE model.
  1. There is a representative household, two firms, and a government.
  2. The government conducts monetary policy and fiscal policy.

- One firm produces the consumption-capital good of the economy with capital, k_t, rented from the household at the real rental rate r_t.

- The other firm creates transaction services, $\mathcal{T}(\cdot)$, the household uses to buy consumption, c_t, and k_t.
  1. $\mathcal{T}(\cdot)$ is an alternative payment technology to fiat currency, M_t,
  2. which is produced by this firm hiring labor services, $\ell_t \in (0, 1)$, from the household at the real wage, w_t.

- G&L construct equilibrium demand functions for k_t and M_t (demand for B_t is implied)
  1. as functions of the household's expectations of the future paths of monetary and fiscal policies and
  2. the nominal interest rate that satisfies arbitrage across the real and nominal assets of the economy.
  3. $\Longrightarrow$ This pins down the relative intertemporal price, r_t, the aggregate price level, P_t, and household portfolio allocations in equilibrium.

G&L: The Household's Problem

- The household chooses uncertain streams of c_t and leisure, $1 - \ell_t$, to maximize its expected lifetime utility, $\mathbf{E}_t\left\{\sum_{t=0}^{\infty}\beta^t\mathcal{U}(c_t,\, 1-\ell_t)\right\}$, subject to budget and transaction medium in advance (TMIA) constraints

$$\begin{aligned} c_t + k_t + \frac{M_t + B_t}{P_t} + P_{\mathcal{T},t}\mathcal{T}_{D,t} &\leq \mathcal{Y}_t + \frac{M_{t-1} + (1+i_{t-1})B_{t-1}}{P_t} + (1-\delta)k_{t-1}, \\ c_t + k_t &\leq \frac{M_{t-1}}{P_t} + \mathcal{T}_{D,t}(c_t + k_t), \end{aligned}$$

  where $\mathcal{U}(\cdot,\,\cdot)$ satisfies the Inada conditions, $\beta,\,\delta \in (0,\,1)$, B_t is a 1-period nominal government bond, $P_{\mathcal{T},t}$ is the relative price (per unit of c_t) of $\mathcal{T}_t$, household income is $\mathcal{Y}_t = w_t\ell_t + (1+r_t)k_t + \mathcal{D}_{G,t} + \mathcal{D}_{\mathcal{T},t}$, and $\mathcal{D}_{j,t}$, $j = G,\,\mathcal{T}$, are dividends the household receives by owning the two firms.

- The budget constraint shows the household faces a portfolio problem $\Longrightarrow$ household allocates saving, $\mathcal{Y}_t - c_t - P_{\mathcal{T},t}\mathcal{T}_{D,t}$, across M_t, B_t, and k_t.

- The CIA constraint limits purchases of c_t and k_t to real balances, M_{t-1}/P_t, and the household's demand for transactions services, $T_{D,t} \in [0,\,1]$.
  1. The household enters date t with M_{t-1} and purchases $T_{D,t}$ at the relative price $P_{\mathcal{T},t}$ before the goods market opens during date t.
  2. When it opens, the household uses M_{t-1} to buy $1 - T_{D,t}$ of $c_t + k_t$.

G&L: The Goods Producing and Transactions Services Firms' Problems

- The goods producing firm chooses k_{t-1} to maximize $\mathcal{D}_{G,t} = (1-\tau_t)f(k_{t-1}) - (1+r_t)k_{t-1}$ by renting it from households,
  1. where τ_t is the proportional tax rate on output of this firm
  2. and the technology, $f(\cdot)$, satisfies $f(0) = 0$ and diminishing marginal returns (DMR), $0 < f'(\cdot)$ and $f''(\cdot) < 0$.
- The transaction services firm hires ℓ_t to maximize $\mathcal{D}_{\mathcal{F},t} = P_{\mathcal{T},t}\mathcal{T}(\ell_t) - w_t\ell_t$, where $\mathcal{T}(0) = 0$ and DMR, $0 < \mathcal{T}'(\cdot)$ and $\mathcal{T}''(\cdot) < 0$, restricts the technology $\mathcal{T}(\cdot)$.
  1. Since $\mathcal{T}_{D,t}$ is an alternative payment mechanism the household substitutes for M_t, the relative price of $\mathcal{T}_t$ to M_t is $P_{\mathcal{T},t}/P_t$.
  2. $\Longrightarrow$ Value of a unit of $\mathcal{T}_t$ multiplied by the purchasing power of money.
  3. As $1/P_t$ falls, $\mathcal{T}_{D,t}$ rises for the household to buy $c_t + k_t \Longrightarrow$ during the Great Inflation of the 1970s, demand for services of money market funds grew in the U.S.

G&L: The Government and Equilibrium

- The government spends g_t in real resources, levies τ_t on the gross output of the goods firm, issues two liabilities, M_t and B_t, and pays i_{t-1} per unit of B_{t-1} to the household $\Longrightarrow$ the government budget constraint is

$$g_t = \tau_t f(k_{t-1}) + \frac{M_t - M_{t-1}}{P_t} + \frac{B_t - (1 + i_{t-1})B_{t-1}}{P_t}.$$

- Besides τ_t, the government's policy instruments are money growth, $\rho_t \equiv M_t / M_{t-1}$, and the government spending-output ratio, $s_{g,t} \equiv g_t / f(k_{t-1})$.
- **Equilibrium**: Given initial conditions $(k_{t-1}, B_{t-1}, i_{t-1})$, prices, monetary policy, and fiscal policy, $\left(P_t, P_{\mathcal{T},t}, i_t, r_{k,t}, w_t, \rho_t, \tau_t, s_{g,t}\right)$, at $t = 1, 2, \ldots,$
  1. the household chooses $(c_t, \ell_t, \mathcal{T}_{D,t}, k_t, M_t, B_t)$ to maximize its expected lifetime utility, subject to budget and TMIA constraints,
  2. the good firm rents k_{t-1} from the household at $r_{k,t} > 0$, and sells $f(k_{t-1})$ to the household for c_t and k_t to maximize $\mathcal{D}_{G,t}$,
  3. the transaction services firms hires ℓ_t from the household at $w_t > 0$ and sells it $\mathcal{T}(\ell_t) = \mathcal{T}_{D,t}$ at $P_{\mathcal{T},t} > 0$ to maximize $\mathcal{D}_{\mathcal{T},t}$,
  4. the money, transaction services, bond, goods, and labor markets clear at non-negative prices $\left(P_t, P_{\mathcal{T},t}, i_t, r_{k,t}, w_t\right)$,
  5. which satisfy the government budget constraint.

G&L: Perfect Foresight and Functional Forms

- Let $\mathcal{Z}_t = (k_t, M_t, B_t)$ and $\mathcal{G}_t = \left\{\mathbf{E}_t\rho_{t+j}, \mathbf{E}_t\tau_{t+j}, \mathbf{E}_t s_{g,t+j}\right\}_{j=0}^{\infty} \Longrightarrow$ endogenous portfolio allocations and exogenous government policy states of the economy at date t.
- Along the equilibrium path, show $\mathcal{Z}_t$ is a function of current and expected future government policies $\Longrightarrow$ map $\mathcal{G}_t$ into household asset choices.
- Up to unanticipated changes in the elements of $\mathcal{G}_t$, G&L study a perfect foresight equilibrium $\Longrightarrow$ there are no exogenous non-policy shocks.
- G&L label the expected discounted values of the future paths of monetary and fiscal policies μ_t and η_t, respectively.
- G&L want closed form or analytic solutions to study the equilibrium responses of $\mathcal{Z}_t$ to changes in μ_t and $\eta_t \Longrightarrow$ choose functional forms
  1. $f(k) = k^{\sigma}, \;\; \sigma \in (0, 1]$,
  2. $\mathcal{T}(\ell) = 1 - (1-\ell)^{\alpha}, \;\; \alpha \in (1, \infty)$, and
  3. $\mathcal{U}(c, 1-\ell) = \ln c + \gamma \ln(1-\ell), \;\; \gamma \in (0, \alpha)$.

The Goods and Transactions Firms' FONCs

- Maximization of the good producing and transactions services firms' profit functions yield the FONCs

$$\begin{aligned} r_t &= (1-\tau_t)\sigma k_{t-1}^{\sigma-1} - 1, \\ \frac{w_t}{P_{\mathcal{T},t}} &= \alpha(1-\ell_t)^{\alpha-1}. \end{aligned}$$

- The optimality condition for the goods producing firm has it demand k_{t-1} at the point where the rental rate of k_{t-1} equals its after-tax marginal product.

- The other firm sets its demand for ℓ_t to equate the real wage relative to the real price of transaction services and the marginal product of labor of transaction services.

The Household's FONCs

- The FONCs with respect to c_t, ℓ_t, $\mathcal{T}_{D,t}$, M_t, B_t, and k_t found by maximizing the household's expected discounted lifetime utility subject to the budget and TMIA constraints are

$$
\begin{aligned}
\varphi_t + \lambda_t(1 - \mathcal{T}_{D,t}) &= c_t^{-1}, \\
\frac{\gamma}{1 - \ell_t} &= \varphi_t w_t, \\
\varphi_t P_{\mathcal{T},t} &= \lambda_t(c_t + k_t), \\
\frac{\varphi_t}{P_t} &= \beta \mathbf{E}_t \left\{ \frac{\varphi_{t+1} + \lambda_{t+1}}{P_{t+1}} \right\}, \\
\frac{\varphi_t}{P_t} &= \beta(1 + i_t)\mathbf{E}_t \left\{ \frac{\varphi_{t+1}}{P_{t+1}} \right\}, \\
\varphi_t + \lambda_t(1 - \mathcal{T}_{D,t}) &= \beta \mathbf{E}_t \left\{ \varphi_{t+1}(1 + r_{t+1}) \right\},
\end{aligned}
$$

where φ_t and λ_t are Lagrange multipliers tied to the household's budget and TMIA constraints and $\delta = 1 \Longrightarrow$ complete depreciation.

Optimality and Equilibrium: Shadow Prices

- Combine the household's FONCs w/r/t ℓ_t and $\mathcal{T}_{D,t}$ to find $\lambda_t = \dfrac{\gamma P_{\mathcal{T},t}}{w_t(1-\ell_t)(c_t+k_t)}$.
  1. Given the transaction services firm's technology is $1-\mathcal{T}_t = (1-\ell_t)^{\alpha}$, this firm's FONC is $w_t(1-\ell_t) = \alpha(1-\mathcal{T}_t)P_{\mathcal{T},t} \Longrightarrow$ the substitution gives $\lambda_t = \dfrac{\gamma}{\alpha}\dfrac{1}{(1-\mathcal{T}_t)(c_t+k_t)}$.
  2. The shadow price of a unit of "liquidity" (*i.e.*, value of a unit of transaction medium = real balances plus transactions services) is less than the value of goods purchased with cash ($\gamma < \alpha$).
  3. $\Longrightarrow$ Trade-off between holding cash, which has a nominal return = 0, and marginal disutility of labor incurred by producing a unit of $\mathcal{T}_t$.

- Substitute for λ_t in the household's FONC for c_t and note in equilibrium $\mathcal{T}_t = \mathcal{T}_{D,t}$ to show $\varphi_t = \dfrac{1}{c_t} - \dfrac{\gamma}{\alpha}\dfrac{1}{(c_t+k_t)}$.
  1. The shadow price of a unit of the good = $MU(c)$ net
  2. of the real resources employed to generate transactions.

Optimality and Equilibrium: Euler Equations and Arbitrage

- The household's Euler equations w/r/t M_t and B_t restrict arbitrage in the markets for these government liabilities.

- The arbitrage restriction is $\mathbf{E}_t \left\{ \dfrac{\lambda_{t+1}}{P_{t+1}} - i_t \dfrac{\varphi_{t+1}}{P_{t+1}} \right\} = 0.$
  1. $\Longrightarrow$ The expected "return" to holding a unit of transaction medium =
  2. expected (real) return to holding a unit of B_t evaluated at net $MU(c_{t+1})$.

- Another arbitrage condition is grounded on the household's Euler equations for B_t and k_t

$$\beta \mathbf{E}_t \left\{ \varphi_{t+1} \left[1 + r_{t+1} - (1 + i_t) \frac{P_t}{P_{t+1}} \right] \right\} \; = \; \lambda_t (1 - \mathcal{T}_t).$$

- There is real wedge between the expected real returns to B_t and k_t.
  1. $\Longrightarrow$ The real wedge is the value of economizing on holding M_t/P_t.
  2. Transaction services are valuable because there is less need to hold M_t, with a real return $= -P_{t+1}/P_t$, to buy goods, but producing $\mathcal{T}_t$ consumes real resources $\Longrightarrow$ household leisure.

Optimality and Equilibrium: Euler Equations and Fiscal Policy, I

- Substitute for φ_t, λ_t, φ_{t+1}, and r_{t+1} in the Euler equation for k_t

$$\frac{1}{c_t} = \beta\sigma\mathbf{E}_t\left\{\frac{c_{t+1} + k_{t+1} - \gamma c_{t+1}/\alpha}{c_{t+1}(c_{t+1} + k_{t+1})}(1 - \tau_{t+1})k_t^{\sigma-1}\right\}.$$

- Since the aggregate resource constraint is $c_t + k_t = (1 - s_{g,t})k_{t-1}^{\sigma}$, the Euler equation becomes

$$\frac{k_t}{c_t} = \beta\sigma\mathbf{E}_t\left\{\left[1 + \frac{k_{t+1}}{c_{t+1}} - \frac{\gamma}{\alpha}\right]\left[\frac{1 - \tau_{t+1}}{1 - s_{g,t+1}}\right]\right\}.$$

- Define the saving rate $sv_t = k_t/(c_t + k_t) \Longrightarrow 1 + k_t/c_t = 1\,/(1 - sv_t)$, which is substituted into the Euler equation to find

$$\frac{1}{1 - sv_t} = \beta\sigma\mathbf{E}_t\left\{\left[\frac{1}{1 - sv_{t+1}}\right]\left[\frac{1 - \tau_{t+1}}{1 - s_{g,t+1}}\right]\right\} + \mathbf{E}_t\left\{1 - \frac{\beta\sigma\gamma}{\alpha}\left[\frac{1 - \tau_{t+1}}{1 - s_{g,t+1}}\right]\right\}.$$

- This forward-looking first-order stochastic difference equation shows household saving decisions are a function of expectations about future fiscal policy or unanticipated changes in τ_{t+j} and $s_{g,t+j}$, $j > 0$.

Optimality and Equilibrium: Euler Equations and Fiscal Policy, II

- G&L conjecture the solution of the forward-looking first-order stochastic difference equation is $1/(1 - sv_t) = \eta_t \Longrightarrow$ iterate the stochastic difference equation forward in η_t, where

$$\eta_t = \mathbf{E}_t\left\{\sum_{j=0}^{\infty}(\beta\sigma)^j d_{\eta,j}\left[1 - \frac{\beta\sigma\gamma}{\alpha}\left[\frac{1-\tau_{t+1+j}}{1-s_{g,t+1+j}}\right]\right]\right\},$$

$$d_{\eta,j} \equiv \prod_{i=0}^{j-1}\left(\frac{1-\tau_{t+1+i}}{1-s_{g,t+1+i}}\right), \text{ and } d_{\eta,0} \equiv 1.$$

- Current savings are a (nonlinear) function of expectations about the relative positions of tax and spending policies from date t to the infinite horizon.
  1. If τ_{t+1+j} ($s_{g,t+1+j}$) falls (rises) all else constant, η_t increases.
  2. The household needs more savings in anticipation of higher future taxes to satisfy the intertemporal government budget constraint.
  3. Note the shape of these responses are restricted by preferences and technologies $\Longrightarrow$ through the structural parameters β, σ, γ, and α.
  4. These responses have a time-varying discount, which moves with anticipated changes in τ_{t+1+j} and $s_{g,t+1+j}$.

Optimality and Equilibrium: Euler Equations and Monetary Policy, I

- A similar procedure results in the equilibrium law of motion of $\mathcal{T}$.
- Putting together the Euler equation for M_t, φ_t, and λ_t yields

$$\frac{1}{P_t}\left[\frac{1}{c_t} - \frac{\gamma/\alpha}{c_t + k_t}\right] = \beta \mathbf{E}_t\left\{\frac{1}{P_{t+1}}\left[\frac{1}{c_{t+1}} - \frac{\gamma/\alpha}{c_{t+1} + k_{t+1}}\left(1 - \frac{1}{1 - \mathcal{T}_{t+1}}\right)\right]\right\}.$$

- Next, substitute for the TMIA constraint, at equilibrium, to show

$$\frac{1 - \mathcal{T}_t}{M_{t-1}}\left[1 + \frac{k_t}{c_t} - \frac{\gamma}{\alpha}\right] = \frac{\beta}{M_t}\mathbf{E}_t\left\{(1 - \mathcal{T}_{t+1})\left[1 + \frac{k_{t+1}}{c_{t+1}} - \frac{\gamma}{\alpha}\right] + \frac{\gamma}{\alpha}\right\}.$$

- Applying the definitions of sv_t and ρ_t gives the Euler equation defining optimality in the money market

$$(1 - \mathcal{T}_t)\left[\frac{1}{1 - sv_t} - \frac{\gamma}{\alpha}\right] = \frac{\beta}{\rho_t}\mathbf{E}_t\left\{(1 - \mathcal{T}_{t+1})\left[\frac{1}{1 - sv_{t+1}} - \frac{\gamma}{\alpha}\right] + \frac{\gamma}{\alpha}\right\}.$$

- This is a first order stochastic difference equation in $\mathcal{T}_t$ and sv_t.
  1. The economy has a recursive solution $\Longrightarrow$ solve for sv_t and then for $\mathcal{T}_t$.
  2. A classical dichotomy exists in which the real and nominal sides of the economy are separated $\Longrightarrow$ solve for marginal value of savings (per unit of consumption), η_t, and next for the marginal value of real balances, μ_t.

Optimality and Equilibrium: Euler Equations and Monetary Policy, II

- G&L iterate forward the stochastic difference equation in $\mathcal{T}_t$ and sv_t to show

$$
\begin{aligned}
(1-\mathcal{T}_t)\left[\frac{1}{1-sv_t}-\frac{\gamma}{\alpha}\right] &= \frac{\mu_t}{\rho_t},\\
\mu_t &= \frac{\beta\gamma}{\alpha}\mathbf{E}_t\sum_{j=0}^{\infty}\beta^j d_{\mu,j},\\
d_{\mu,j} &\equiv \prod_{i=0}^{j-1}\left(\frac{1}{\rho_{t+1+j}}\right), \text{ and } d_{\mu,0} \equiv 1.
\end{aligned}
$$

- Given sv_t, the fraction of transactions conducted in real balances is a function of the expected discounted future path of money growth.
  1. If money growth is expected to increase in the future, the fraction of transactions conducted in cash falls today.
  2. Note the shape of response of the household's choice of transaction medium is restricted by the structural parameters β, σ, γ, and α.
  3. These responses have a time-varying discount, which inversely with anticipated changes in ρ_{t+1+j}.

Optimality and Equilibrium: Goods and Money Markets

- Market clearing in the goods market and the definition of sv_t yields
  1. the consumption function, $c_t = (1 - s_{g,t})k^{\sigma}_{t-1}/\eta_t$.
  2. The flip side of the consumption function is the equilibrium law of motion of capital, $k_t = \left[1 - \dfrac{1 - s_{g,t}}{\eta_t}\right]k^{\sigma}_{t-1}$.
  3. Along the equilibrium path, c_t and k_t are fractions of current output, where movements in these fractions are driven by expectations of the future path of fiscal policy.

- The money demand schedule is found by combining the aggregate resource and the TMIA constraints and the definitions of η_t and μ_t to find

$$\frac{M_t}{P_t} = \left[\frac{\mu_t}{\eta_t - \gamma/\alpha}\right](1 - s_{g,t})k^{\sigma}_{t-1}.$$

- Money demand and price determination depend on anticipated future monetary and fiscal policies.

Optimality and Equilibrium: Goods and the Money Market, I

- An arbitrage condition restricts household choices of the nominal assets.
- Start with the bond Euler equation, $\varphi_t/P_t = \beta(1+i_t)\mathbf{E}_t\left\{\varphi_{t+1}/P_{t+1}\right\}$, and $\varphi_t = 1/c_t - \gamma/\left[\alpha(c_t+k_t)\right]$ to show
$$\frac{1}{P_t(c_t+k_t)}\left[1+\frac{k_t}{c_t}-\frac{\gamma}{\alpha}\right] = \beta(1+i_t)\mathbf{E}_t\left\{\frac{1}{P_{t+1}(c_{t+1}+k_{t+1})}\left[1+\frac{k_{t+1}}{c_{t+1}}-\frac{\gamma}{\alpha}\right]\right\}.$$
- Substitute the (equilibrium) TMIA constraint, $P_t(c_t+k_t) = M_{t-1}/(1-\mathcal{T}_t)$, and the definition of sv_t into the bond Euler equation to obtain
$$(1-\mathcal{T}_t)\left[\frac{1}{1-sv_t}-\frac{\gamma}{\alpha}\right] = \beta(1+i_t)\frac{M_{t-1}}{M_t}\mathbf{E}_t\left\{(1-\mathcal{T}_{t+1})\left[\frac{1}{1-sv_{t+1}}-\frac{\gamma}{\alpha}\right]\right\}.$$
- Since $\rho_t = M_t/M_{t-1}$, the Euler equations for money and bonds yield
$$\begin{aligned}\frac{\gamma}{\alpha} &= i_t\mathbf{E}_t\left\{(1-\mathcal{T}_{t+1})\left[\frac{1}{1-sv_{t+1}}-\frac{\gamma}{\alpha}\right]\right\},\\ \frac{\gamma}{\alpha} &= \frac{\rho_t i_t}{\beta(1+i_t)}(1-\mathcal{T}_t)\left[\frac{1}{1-sv_t}-\frac{\gamma}{\alpha}\right],\\ \frac{\beta\gamma}{\alpha}\left(1+i_t^{-1}\right) &= \rho_t(1-\mathcal{T}_t)\left[\frac{1}{1-sv_t}-\frac{\gamma}{\alpha}\right].\end{aligned}$$

Optimality and Equilibrium: Goods and the Money Market, II

- Imposing equilibrium in the money market produces the arbitrage condition

$$\mu_t = \frac{\beta\gamma}{\alpha}\left(1 + \frac{1}{i_t}\right) \Leftrightarrow i_t = \frac{\beta\gamma}{\alpha\mu_t - \beta\gamma}, \text{ where } i_t \in (0, 1).$$

- In the G&L model, arbitrage restricts the nominal return on the government bond to be a nonlinear function of the expected path of money growth.
- Substitute for μ_t in the equilibrium condition for real balances to create a "money demand function"

$$\frac{M_t}{P_t} = \frac{\beta}{\alpha\eta_t/\gamma - 1}\left(1 + \frac{1}{i_t}\right)(1 - s_{g,t})k^{\sigma}_{t-1}.$$

- The G&L money demand function has income and interest rate elasticities with standard signs, but current and expectations about future fiscal policies also drive household decisions about real balances.
  1. This is a non-standard result because it violates the classical dichotomy.
  2. $\Longrightarrow$ Price level determination depends on B_t through anticipated changes in spending and tax policies.
  3. Nonetheless, i_t varies only with expectations about future money growth.

Policy for a Stationary Equilibrium and Policy Analysis

- G&L conduct policy analysis with their model by assuming $\rho_{t+j} = \overline{\rho}$, $\tau_{t+j} = \overline{\tau}$, and $s_{g,t+j} = \overline{s}_g$, for $j \geq 0$, which produces a stationary equilibrium.
- $\Longrightarrow$ Analyze deviations from the stationary equilibrium generated by a change in policy at date $t+h$.
- Under $\rho_{t+j} = \overline{\rho}$, $\tau_{t+j} = \overline{\tau}$, and $s_{g,t+j} = \overline{s}_g$, for $j \geq 0$, the stationary equilibrium relies on the policy functions

$$\eta_t\left(\overline{\tau}, \overline{s}_g\right) = \frac{\alpha(1-\overline{s}_g) - \beta\sigma\gamma(1-\overline{\tau})}{\alpha[1-\overline{s}_g - \beta\sigma(1-\overline{\tau})]} \quad \text{and} \quad \mu_t(\overline{\rho}) = \frac{\beta\gamma\overline{\rho}}{\alpha(\overline{\rho}-\beta)},$$

where the signs of the policy response functions are

$$\frac{\partial\eta_t\left(\overline{\tau}, \overline{s}_g\right)}{\partial\overline{\tau}} = \frac{\beta\sigma(\gamma-\alpha)(1-\overline{s}_g)}{\alpha[1-\overline{s}_g-\beta\sigma(1-\overline{\tau})]^2} < 0,$$

$$\frac{\partial\eta_t\left(\overline{\tau}, \overline{s}_g\right)}{\partial\overline{s}_g} = \frac{\beta\sigma(\alpha-\gamma)(1-\overline{\tau})}{\alpha[1-\overline{s}_g-\beta\sigma(1-\overline{\tau})]^2} > 0,$$

$$\frac{\partial\mu_t(\overline{\rho})}{\partial\overline{\rho}} = \frac{-\beta^2\gamma}{\alpha[\overline{\rho}-\beta]^2} < 0.$$

Equilibrium Price Determination: The Tobin Effect

- The partial derivatives aid in characterizing the equilibrium price level.
- Define $\mathcal{V}_t \equiv \mu_t/[\eta_t - \gamma/\alpha]$, where $\mathcal{V}_t$ denotes velocity, $M_t/[P_t(1-s_{g,t})k^{\sigma}_{t-1}]$, and substituting for η_t and μ_t at $\rho_{t+j} = \overline{\rho}$, $\tau_{t+j} = \overline{\tau}$, and $s_{g,t+j} = \overline{s}_g$, for $j \geq 0$, gives

$$\mathcal{V}_t\left(\overline{\rho}, \overline{\tau}, \overline{s}_g\right) = \frac{\beta\gamma}{\alpha(\alpha-\gamma)}\left[1 + \beta\sigma\frac{(1-\overline{\tau})}{(1-\overline{s}_g)}\right]\frac{\overline{\rho}}{\overline{\rho}-\beta}.$$

- The derivatives of $\mathcal{V}_t$ are $\dfrac{\partial\mathcal{V}_t\left(\overline{\rho}, \overline{\tau}, \overline{s}_g\right)}{\partial\overline{\rho}} = \dfrac{-\beta\mathcal{V}_t\left(\overline{\rho}, \overline{\tau}, \overline{s}_g\right)}{\overline{\rho}(\overline{\rho}-\beta)} < 0$, $\dfrac{\partial\mathcal{V}_t\left(\overline{\rho}, \overline{\tau}, \overline{s}_g\right)}{\partial\overline{\tau}} = \dfrac{\beta^2\sigma\gamma}{\alpha(\alpha-\sigma)(1-\overline{s}_g)} > 0$, and $\dfrac{\partial\mathcal{V}_t\left(\overline{\rho}, \overline{\tau}, \overline{s}_g\right)}{\partial\overline{s}_g} = \dfrac{-\beta^2\sigma\gamma}{\alpha(\alpha-\sigma)}\dfrac{(1-\overline{\tau})}{(1-\overline{s}_g)^2} < 0.$
- The last two derivatives are non-zero suggesting anticipated changes in fiscal policy affect the equilibrium price level.
- The household and firms expect changes in $\overline{\rho}$, $\overline{\tau}$, or $\overline{s}_g$ to induce movements in i_t, M_t, and/or B_t, along with r_t, k_t, and/or c_t.
  1. $\Longrightarrow$ Fiscal policies alter the equilibrium price level because the government budget constraint and arbitrage must be satisfied along the equilibrium path.
  2. The Tobin effect predicts anticipated movements in the aggregate price level (or expected inflation) has real effects.

Equilibrium Price Determination: Anticipated Changes in Policy

- The three derivatives of $\mathcal{V}_t$ describe mechanisms through which expectations about monetary policy and fiscal policy affect the equilibrium price level.

- Anticipated changes in ρ_{t+j} alter μ_t, which shows monetary policy generates movements in the marginal value of a unit of real balances.
  1. **Experiment**: Increase some ρ_{t+j}s $\Longrightarrow$ $\mu_t(\overline{\rho})$ falls.
  2. The G&L model predicts the purchasing power of M_t falls or P_t rises.
  3. Arbitrage demands the household shift out of cash either to k_t or B_t.

- Tax and spending policy experiments are embedded in $\eta(\overline{\tau}, \overline{s}_g)$.
  1. **Experiment**: Raise some τ_{t+j}s $\Longrightarrow$ $\eta(\overline{\tau}, \overline{s}_g)$ drops. (Assume $B_t = 0$ and tax revenue is rebated lump sum to the goods firm.)
  2. The household demands more c_t and M_t and holds less k_t because its future returns are expected to fall.
  3. Given $M_t = \overline{M}$, increased in M_t forces P_t to fall $\Longrightarrow$ real balances rise.
  4. **Experiment**: Increase some government budget deficits, $g_t - \tau_{t+j}k^{\sigma}_{t-1+j}$.
  5. Expectations of whether the government satisfies its budget constraint by collecting real (*i.e.*, tax capital) or nominal (*i.e.*, seigniorage) revenue matters for the determining the equilibrium price level.

Equilibrium Price Determination: The Government Deficit

- The Tobin effect works (in part) through the government budget constraint, which can be written as

$$\frac{1-\overline{\tau}}{1-\overline{s}_g} = 1 + \mathcal{V}_t\left[\frac{\rho_t - 1}{\rho_t} + \left(\frac{B_t}{M_t} - \frac{1+i_t}{\rho_t}\frac{B_{t-1}}{M_{t-1}}\right)\right].$$

- The ratio to the left of the equality define the government demand for real resources to finance its budget deficit.
  1. Demand by the government for goods owned by the household or goods firm,
  2. which is a Tobin effect lacking any impact on i_t.
  3. $\Longrightarrow$ Equilibrium price determinations sans nominal factors or outside of conventional money demand or interest rate rule environments.

Equilibrium Price Determination: Government Revenue Sources

- The Tobin effect suggests there are trade-offs to financing the government deficit with B_t or M_t.
- This trade-off is analyzed by rewriting the previous government budget to show the explicit demand placed on the household and goods firm to pay for the government deficit, $\overline{s}_g - \overline{\tau}$, per unit of private resources, $1 - \overline{s}_g$,

$$\frac{\overline{s}_g - \overline{\tau}}{1 - \overline{s}_g} = \mathcal{V}_t \left[\frac{\rho_t - 1}{\rho_t} + \left(\frac{B_t}{M_t} - \frac{1 + i_t}{\rho_t} \frac{B_{t-1}}{M_{t-1}} \right) \right].$$

- First term in brackets to the right of the equality represents seigniorage revenue.
- The next two terms reflect the revenue raised by issuing new nominal government debt net of interest and principle payments on debt being retired by the government.
  1. However, this government budget constraint divides nominal government debt by fiat currency and multiplies by the velocity of money.
  2. Along the equilibrium path changes in the stock of government debt (fiat currency) to finance the deficit generates changes in the demand for fiat currency (government debt) and the relative price of these nominal liabilities.
  3. $\Longrightarrow$ Current fiscal and monetary policies may have to adjust to anticipated shifts in these policies in the future to satisfy the government budget constraint.

G&L: Summary

- G&L teach a lesson that expected changes in monetary (fiscal) policy cannot be studied in isolation of fiscal (monetary) policy.
- There are several ways to motivate these policy interactions.
  1. Arbitrage connects nominal and real household portfolio decisions.
  2. The opportunity cost of holding cash is quantified by i_t and expectations of the path of fiscal policy $\Longrightarrow$ money demand is not only about income and interest elasticities.
  3. Changes in future monetary and fiscal policies require the current government budget constraint to hold along the equilibrium path.
- Standard results in macroeconomics rely on a slew of unstated assumptions about monetary policy and fiscal policy:
  1. quantity theory, money demand and price level determination, new Keynesian policy prescriptions,
  2. open market operations, and the fiscal theory of the price level.
- Monetary and fiscal policy interactions brings the mechanism defined by the Tobin effect to the center of macro analysis.

Are FIs Necessary and/or Sufficient? To Do What?

- The LF and G&L models suggest two questions.
  1. Is the FI providing services to the economy that the household cannot when it engages in direct asset trades with the firm?
  2. What financial services matter most for the monetary transmission mechanism?
- FIs evaluate and monitor loans, which is a service thought essential for efficient transfer of household saving to firms.
- Why not have households evaluate and monitor loans instead of delegating to FIs the job of producing these services?
- Households could, but FIs take on these roles in practice.
  1. Are there frictions in financial markets that are mitigated by delegating loan evaluation and monitoring to FIs?
  2. Is it more efficient to have FIs take on these activities?
  3. What conditions/assumptions are necessary and/or sufficient for an economy to obtain these efficiency gains?

Williamson (JPE, 1987)

- Williamson studies a financial friction in a rational expectations DSGE model that generates FIs endogenously.
- The financial friction is loan default (*i.e.*, bankruptcy costs).
- Borrowers have incentives to default on loans because they know their productivity, but no one else does.
- The incentive is that a borrower receiving a low productivity realization may want "to take the money and run."
- The economy delegates loan evaluation and monitoring to coalitions of households who form FIs
    1. to minimize the costs associated with loan defaults
    2. by imposing an incentive compatibility constraint (ICC) on borrowers to make them better off fulfilling the debt contract than consuming their loans.
- Thus, FIs are an endogenous response to the loan default friction.

Asymmetric Information and Debt Contracts

- A loan default friction occurs because FIs do not see the productivity of borrowers.
- The asymmetric information problem is minimized by FIs offering borrowers a debt contract that is an ICC.
- The ICC induces borrowers to "tell the truth" about their productivity draws.
- Borrowers can do no better in welfare terms by telling the truth about their productivity draws when facing the ICC embedded in the debt contract offered by the FI.
- Thus, the FI's debt contract is an optimal response to borrowers not being able to credibly reveal their productivity realization.

Adverse Selection and Moral Hazard

- Williamson invokes asymmetric information, but it does not create an adverse selection environment.
  1. FIs are not facing a risk of only loaning to high cost-high default rate borrowers.
  2. Rather ex ante the FI lends to borrowers for whom the productivity draw is realized in the future (only after the debt contract is struck).

- Moral hazard arises in models of asymmetric information, but there is difference between ex post and ex ante moral hazard.
  1. Ex ante moral hazard is about an agent adopting a policy rule that induce more risk in outcomes.
  2. Ex post moral hazard is driven by incentives to make matters worse after a bad event is realized.

- Williamson works with ex post moral hazard in a model of asymmetric information and costly state verification.

Williamson's Model: Households

- The over-lapping generations (OLG) model is employed to have
  1. households that are heterogeneous across time,
  2. endogenously generated inside "money" and
  3. fiat currency that has strictly positive value.
- Households maximizes utility over their two period lives.
  1. A young household chooses consumption, c_y, and leisure, ℓ_y.
  2. Consumption, c_o, is the only choice when old.
  3. Consumption and leisure choices yield ex ante uncertain outcomes for households.
- Utility is time-separable, $\mathcal{U}(c_y, \ell_y, c_o) = v(c_y, \ell_y) + c_o$, where
  1. $v_j > 0$ and $v_{jj} < 0,\ j = c, \ell$, and
  2. c and ℓ are normal goods $\Longrightarrow v_{jj} - v_{kj} < 0$.
- The economy does not grow.
  1. There is always one old household per young household.

Williamson's Model: Household Types

- Households born at date t, where $t = 1, 2, \ldots, \infty$.
  1. receive an address, $i = 1, 2, \ldots, \infty$, that is drawn from
  2. the countably infinite set of addresses $\Omega = \{1, 2, \ldots, \infty\}$,
  3. which defines household type, $d_i = 1, 2$.
- Type 1 households consume when young and old.
  1. They receive an endowment of h_y $(= 1 - \ell_y)$ units of labor
  2. that is transformed one-for-one into the economy's single perishable consumption good.
- The ith type 2 household consumes only when old by
  1. operating a technology during date t that
  2. needs an input of one unit of c_t to produce date $t+1$ output ω_i,
  3. which is stochastic, strictly positive, where $\mathbf{E}\{\omega_i\} > 1$, $\forall\ i$, and
  4. ω_i is private information of this agent.
- A date $t = 1$ initial condition is that a household of the initial old generation is endowed with H units of fiat currency.

Williamson's Model: Technologies

- The output of the production technology yields a return that
  1. is *i.i.d.* across all i type 2 households of generation t,
  2. drawn from the pdf $f(\cdot)$ with the associated distribution $\mathcal{F}(\cdot)$,
  3. where $f(\cdot)$, which is observed by all households, is differentiable on the real line $[0, \overline{\omega}]$.
- When the ith type 1 household tries to monitor date $t+1$ output of a type 2 household, the cost is γ_i units of consumption, $\gamma_i \in (0, \infty)$.
- A countably infinite number of household's address?
  1. The probability of type i households on $[1, n]$ is $\mathcal{P}_n(\mathcal{A}) = \dfrac{1}{n}$.
  2. As $n \longrightarrow \infty$, define $\mathcal{D}(\mathcal{A}) = \mathcal{P}_n(\mathcal{A})$.
  3. Let $g(\cdot)$ be a pdf $\in (0, \infty)$ of the distribution $\mathcal{G}(\cdot)$.
  4. $\mathcal{D}(d_i = 1 \,|\, \gamma \leq \overline{\gamma}) = \alpha\mathcal{G}(\overline{\gamma})$ and $\mathcal{D}(d_i = 2 \,|\, \gamma \leq \overline{\gamma}) = (1-\alpha)\mathcal{G}(\overline{\gamma})$
  5. The 2-tuple (d_i, γ_i) is known to all agents in the economy.

Williamson's Model: Market Structure

- All markets are perfectly competitive.
  1. Households are price takers.
  2. The economy is large compared to a household.
- Young type 1 households can either lend to type 2 households or purchase fiat currency from the old to intertemporally substitute consumption between dates t and $t+1$.
- Young type 2 households produce, but lack resources to run $f(\cdot)$.
  1. The device these households use to substitute intertemporally,
  2. but this occurs only by borrowing from type 1 households.
- However, type 1 households need to monitor type 2 households.
- Otherwise, the ith type 2 household might abscond with ω_i if the draw from $f(\cdot)$ is low.

Williamson's Model: Generating FIs

- Given $h_y < 1$, several young type 1 households must combine to lend a unit of the consumption good to a type 2 household.
- These collections of type 1 households are FIs.
  1. A FI offers the young type 1 households a deposit contract for their consumption good.
  2. A unit of the consumption good is lent by the FI to a type 2 household at date t in a competitive credit market.
  3. A loan's monitoring costs is the γ_i of the type 2 household receiving the loan from the FI.
  4. The credit market clears at the rate r_t, which Williamson calls the "market expected return".
  5. The rate is expected because it clears the credit market prior to the ω_is being realized in the next period.

Williamson's Model: The Optimal Loan Contract

- A date t loan contract is optimal *iff* it minimizes monitoring costs.
  1. The contract extracts a commitment from the ith type 2 household of re-payment of x_i, which is public information.
  2. Assume monitoring is not random, but occurs when $\omega_i < x_i$.
  3. Otherwise, $\omega_i \geq x_i \implies$ the ith type 2 agent fulfills the contract.
- Note that when monitoring occurs, the FI receives a negative repayment because $\omega_i < x_i$.
- Thus, Williamson argues that x_i is the interest payment the FI charges the ith type 2 household.
- Remember the assumption is $\mathbf{E}\{\omega_i\} > 1, \forall\ i$, which implies the same is true for x_i.

Williamson's Model: The Optimal Loan Contract Problem

- A ith type 2 household wants to maximize the expected return to operating its production technology at date t.
  1. Output is ω_i, which generates a return $f(\omega_i)$.
  2. The loan contract re-payment is a definite commitment of x_i.
  3. Therefore, the objective is $\max_{x_i} \int_x^{\omega_i} (\omega_i - x) f(\omega_i) d\omega$.
- This objective is constrained by the FI's need to earn r_t on its loan.
  1. Monitoring of the loan garners the FI $\int_0^x (\omega_i - \gamma_i) f(\omega_i) d\omega$.
  2. Otherwise, a FI receives $x_i \left[1 - \mathcal{F}(x_i)\right]$ because $\mathcal{F}(x_i)$, which is the income from producing realized by the ith type 2 household, nets against the one unit loan of c_t.
  3. Thus, the ith type 2 household's objective is subject to

$$\int_0^x (\omega_i - \gamma_i) f(\omega_i) d\omega \; + \; x_i[1 - \mathcal{F}(x_i)] \; \leq \; r_t.$$

Williamson's Model: The Equilibrium Return on the Loan

- The return to the ith type 2 household's technology, $\mathcal{R}_i$, is
  1. x_i when $x_i \leq \omega_i$ or
  2. ω_i when $x_i > \omega_i$.
- Suppose a FI offers $\mathcal{M}$ ($< \infty$) type 2 households the optimal loan contract.
- The expected return, $\mathbf{E}\{\mathcal{R}_i\}$, on these loans is the average of the $\mathcal{R}_i$s, $i = 1, 2, \ldots, \mathcal{M}$, or $\frac{1}{\mathcal{M}} \sum_{i=1}^{\mathcal{M}} \mathcal{R}_i$.
- In equilibrium, the FI offers the optimal loan contract to all type 2 households in the perfectly competitive credit market.
- Since the number of type 2 households is countably infinite, the expected return on a FI's loan portfolio is computed as $\mathcal{M} \longrightarrow \infty$.
- Thus, the date t expected return on a FI's loan portfolio is the probability limit of $\frac{1}{\mathcal{M}} \sum_{i=1}^{\mathcal{M}} \mathcal{R}_i$, or $\mathbf{E}\{\mathcal{R}_i\} = r_t$ in equilibrium.

Williamson's Model: The FI in Equilibrium

- There is one and only one FI in equilibrium.
  1. Only one FI $\Longrightarrow$ need to diversify borrower monitoring costs.
  2. This spreads the (potential) cost of default across a large, a very large, number of loans or $\lim_{\mathcal{M}\to\infty} \dfrac{1}{\mathcal{M}} \sum_{i=1}^{\mathcal{M}} \gamma_i = 0$.
  3. A single lender also eliminates the replication of monitoring costs when households borrow and lend with each other.
  4. FIs need a large mass of borrowers to diversify monitoring costs $\Longrightarrow$ FIs fail only given a large negative aggregate shock.

- The single FI earns zero profits subsequent to paying type 1 households the gross return on their deposits.
  1. There are a countably infinite number of depositors, which drives monitoring costs to zero $\Longrightarrow$ deposits pay a certain r_t in equilibrium.
  2. The return on deposits is certain because diversification spreads out these costs across a large number of loans.

Williamson's Model: The FI in Practice

- Deposit and loan contracts differ.
  1. In exchange for some c_t, the FI offers type 1 households a one-period deposit contract with a guaranteed return between dates t and $t+1$.
  2. Type 2 households sign a loan contract that commits them to return at least date t production to the FI.
  3. Loan payoff is uncertain ex ante, but return on a FI's loan book is not.

- Ex post the FI knows more about the economy than do households.
  1. Type 1 households never know which type 2 agents default and the bankruptcy cost such an event imposes on the economy.
  2. Explains the role diversification plays in the economy, but type 1 and type 2 households are risk neutral over their consumption when old.

Williamson's Model: The FI and Diversification

- The FI's expected return on a loan to the ith type 2 household is

$$\Pi(x_i,\, y_i) \;=\; \int_0^x (\omega_i - y_i) f(\omega_i) d\omega \;+\; x_i[1 - \mathcal{F}(x_i)].$$

- Note $\int_0^x (\omega_i - y_i) f(\omega_i) d\omega = \int_0^x \omega_i \mathcal{F}'(\omega_i) d\omega - y_i \int_0^x \mathcal{F}'(\omega_i) d\omega$.
- Integration by parts, $\int a(z) b'(z) dz = a(z) b(z) - \int a'(z) b(z) dz$, results in

$$\begin{aligned} \int_0^x \omega_i \mathcal{F}'(\omega_i) d\omega \quad &= \quad \omega_i \mathcal{F}(\omega_i) \Big|_0^x \;-\; \int_0^x \mathcal{F}(\omega_i) d\omega \\ &= \quad x_i \mathcal{F}(x_i) \;-\; \int_0^x \mathcal{F}(\omega_i) d\omega. \end{aligned}$$

Williamson's Model: The FI and Diversification, cont.

- The FI's expected return on a loan to the ith type 2 household is

$$\begin{aligned}\Pi(x_i, y_i) &= \int_0^x (\omega_i - y_i) f(\omega_i) d\omega + x_i[1 - \mathcal{F}(x_i)] \\ &= x_i \mathcal{F}(x_i) - \int_0^x \mathcal{F}(\omega_i) d\omega - y_i \int_0^x \mathcal{F}'(\omega_i) d\omega \\ &\qquad + x_i[1 - \mathcal{F}(x_i)] \\ &= x_i - \left[\int_0^x \mathcal{F}(\omega_i) d\omega + y_i \mathcal{F}(x_i)\right]\end{aligned}$$

- The expected return on a loan is the committed payment of x_i by the ith type 2 household and a risk premium.

Williamson's Model: The FI and Diversification, cont.

- The risk premium is $-\Big[\int_0^x \mathcal{F}(\omega_i)d\omega \,+\, \gamma_i\mathcal{F}(x_i)\Big]$.
- The expected loan return is lower because the FI faces a risk the type 2 households may not repay the loan.
- The loan's risk increases as the risk on the project's returns rise, according to first-order stochastic dominance.
  1. Suppose that $\mathcal{F}(\cdot)$ represents a less risky set of random project returns than $\mathbb{F}(\cdot)$.
  2. First-order stochastic dominance predicts $\mathcal{F}(\omega) \geq \mathbb{F}(\omega)$, for every $\omega \in [0, \overline{\omega}]$,
  3. and $\mathcal{F}(\omega) > \mathbb{F}(\omega)$ for at least some of these ωs.
- Monitoring costs also create risk for a loan to a type 2 household $\Longrightarrow$ costly to verify the state/type of the borrower.

Williamson's Model: The FI and Diversification, cont.

- The expected return $\Pi(x_i, y_i)$ can be negative for the FI. ($\Longrightarrow$ What is the outcome of a default?)
- Hence, $\dfrac{\partial \Pi(x_i, y_i)}{\partial x_i} \gtreqless 0$, or $\Pi(x_i, y_i)$ is not monotone increasing in x.
- Show that $\Pi(x_i, y_i) = x_i - \left[\int_0^x \mathcal{F}(\omega_i)d\omega + y_i\mathcal{F}(x_i)\right]$ results in $\dfrac{\partial \Pi(x_i, y_i)}{\partial x_i} = 1 - [\mathcal{F}(x_i) + y_i f(x_i)] \ngeq 0$, for some $x_i \in [0, \overline{\omega}]$.
- Williamson assumes that when $\Pi(x_i, y_i)$ is non-negative,
  1. $\dfrac{\partial^2 \Pi(x_i, y_i)}{\partial x_i^2} = f(x_i) + y_i f'(x_i)$ is strictly positive, which
  2. restricts the expected return function to be convex in x.
  3. As the x_i charged to the ith type 2 household increases, the FI's expects its return on the loan to decrease at an increasing rate.

Williamson's Model: The FI and Diversification, cont.

- The expected return $\Pi(x_i, y_i)$ can be negative for the FI. ($\Longrightarrow$ What is the implication of a large y_i?)
- Integrate out the debt payment to which the borrower commits to show a large FI's expected loan return is $\Pi(y_i) = \max_{x_i} \Pi(x_i, y_i)$.
- Apply the envelope condition to the positive segment of $\Pi(y_i)$, which is $\dfrac{d\Pi(x_i, y_i)}{dy_i} < 0$ (remember $\dfrac{\partial\Pi(x_i, y_i)}{\partial y_i} = -\mathcal{F}(x_i) < 0$).
- A loan's expected return is a decreasing function of y_i.
- Thus, the loan's expected return is completely characterized.
  1. The FI is constrained from demanding "too large" of a pre-commitment payment from the borrower.
  2. Otherwise, only the riskiest type 2 households borrow to engage in production.

BGG's Antecedents: Relevant Literature

- Irving Fisher (1933) was a proto-monetarist and the Reinhart and Rogoff (2011) of the Great Depression $\Longrightarrow$ price deflation (aggregate or asset?) is observed causal shock.
- Keynes' General Theory (1936) is about multiple equilibria.
- The sources of the General Theory's multiple equilibria are
  1. households are off their labor supply and saving schedules.
  2. Aggregate and relative price movements do not generate real allocations that keep the economy at full employment $\Longrightarrow$ insufficient aggregate demand.
- Bernanke (1983) argues the Great Depression was induced by a financial friction and a financial shock, which differs from the monetary shock story told by Friedman and Schwartz (1961).
- Financial market frictions suggest financial shocks operate on aggregate demand generating Keynesian business cycle stories.

Bernanke, Gertler, and Gilchrist (1998)

- BGG aim to build a rational expectations DSGE model that violates the Miller and Modigliani (M&M) theorem.
- M&M theorem states the value of a firm is unaffected by whether it is financed by debt or equity, assumes markets are competitive and efficient.
- Given no frictions, the firm operates in a Arrow-Debreu economy.
    1. There exist a complete set of state contingent securities.
    2. These securities let agents insure against or hedge all risk.
- The frictions that are assumed away are, for example, the cost of monitoring agents delegated with management tasks, bankruptcy costs, taxes, collateral constraints, and asymmetric information.

BGG Objectives

- Graft credit market frictions into a DSGE model to
  1. conduct policy evaluations, and
  2. use it as a forecasting tool.
- Show credit market frictions in a DSGE model match business cycle
  1. volatility,
  2. persistence,
  3. and comovement.
- BGG-DSGE models depend on a propagation mechanism that operates on aggregate demand not aggregate supply.
- The propagation mechanism relies on asymmetric information.
  1. BGG are motivated by an agency or principal-agent problem.
  2. Williamson works in the tradition of costly state verification, which is a contract design problem.

The Financial Accelerator

- The financial accelerator consists of
  1. "endogenous developments in credit markets (that) work to propagate and amplify shocks to the macroeconomy."
  2. The "endogenous developments" are traced back to the "external finance premium" and the net worth of borrowers.
- The external finance premium is the difference between a firm's
  1. external costs of funding projects, and
  2. the opportunity cost of raising investment funds internally.
- The net worth of borrowers equals
  1. the liquidity value of their assets plus
  2. the collateral value of their less liquid assets
  3. net of their liabilities.
- BGG argue the external finance premium is approximately the inverse of the borrower's net worth.

The BGG–NKDSGE Model: Structure

- The economy consists of heterogenous agents with endogenous financial contracts that can be aggregated into a DSGE model.
- There are entrepreneurs, households, retailers, and a government.
- Households are infinitely lived dynasties that (i) supply labor to intermediate good producers (*i.e.*, entrepreneurs), (ii) demand output from final goods producers, which is either (iii) consumed or (iv) substituted intertemporally using cash and deposits at FIs.
- Final goods producers compete in monopolistic markets and are subject to Calvo (time-dependent) staggered price setting.
- The government runs fiscal policy and monetary policy.
    1. Exogenous government spending.
    2. Lump sum taxation of households.
    3. Issue fiat currency.
    4. A balanced budget period by period.

The BGG Model: Entrepreneurs

- BGG solve the FI's problem in partial equilibrium by taking as given (i) the ex post expected return to capital, (ii) the ex ante value of capital goods, (iii) an entrepreneur's ex ante loan demand.

- Capital is bought by entrepreneurs to produce an intermediate good, which they finance using borrowed external funds.
    1. Capital is not accumulated by entrepreneurs.
    2. They only own the production technology.
    3. Labor is also purchased by entrepreneurs to produce.

- BGG assume that capital is homogenous.
    1. Thus, entrepreneurs do not care if they use capital today that was employed by a competitor in the past.
    2. $\Longrightarrow$ Entrepreneurs buy entire capital stock period by period.
    3. However, they face a net worth restriction on more than just their purchase of new capital (*i.e.*, gross investment).
    4. The restriction is on an entrepreneur's capital stock.

The BGG Model: The Entrepreneur's Return to Capital

- At the end of date t, the jth entrepreneur buys capital, $K_{j,t+1}$.
  1. $K_{j,t+1}$ helps to produce the intermediate good during date $t+1$.
  2. The intermediate good is produced using a CRS technology that also requires a labor input.
- The market price of capital is Q_t (per unit) during date t.
- The ex post gross return on $K_{j,t+1}$ is, $\omega_j R_{K,t+1}$.
  1. ω_j is an idiosyncratic shock, which is $i.i.d.$ drawing from the continuous pdf $f(\cdot)$ with the associated cdf $\mathcal{F}(\cdot)$.
  2. BGG assume $\mathbf{E}\{\omega_j\} = 1$ and $\mathcal{F}(\cdot)$ has non-negative support.
- Thus, the aggregate ex post return to capital is $R_{K,t+1}$ (because this is a partial equilibrium problem).

The BGG Model: The Entrepreneur's Idiosyncratic Shock

- What is the probability the jth entrepreneur receives the idiosyncratic shock $\widetilde{\omega}_j$ on the return to $K_{j,t}$, given that the shock is greater than or equal to x?
    1. The answer is the hazard of $\mathcal{F}(\cdot)$ is $h(\cdot) = \dfrac{f(\cdot)}{1 - \mathcal{F}(\cdot)}$.
    2. $h(\omega)$ measures the instantaneous probability of drawing within an ϵ–neighborhood of x, given $x \leq \widetilde{\omega}_j$.
    3. BGG assume $\dfrac{\partial \omega h(\omega)}{\partial \omega} > 0$, which places a weak restriction on the elasticity of the hazard, $\dfrac{\omega}{h(\omega)} \dfrac{\partial h(\omega)}{\partial \omega} > -1$.
- Perhaps, more useful for analysis of the BGG model is the "survivor function," $1 - \mathcal{F}(\cdot)$.
    1. The survivor function quantifies the probability $x < \widetilde{\omega}_j$.
    2. This probability is information about whether the entrepreneur's debt obligation will be repaid.

The BGG Model: The Entrepreneur's Debt

- BGG assume (i) the entrepreneur is risk neutral and (ii) ex ante the net worth of the jth entrepreneur, $N_{j,t+1}$, is insufficient to purchase $Q_tK_{j,t+1}$ at the end of date t.
- Entrepreneur j borrows $B_{j,t+1} = Q_tK_{j,t+1} - N_{j,t+1}$ from a FI at end of date t
  1. and accepts any and all risk of the debt $B_{j,t+1}$.
  2. Why? $\Longrightarrow$ risk neutral FIs diversify away idiosyncratic risk of default by entrepreneurs instead of risk averse households demanding a premium to do so.
- The realization of ω_j is the jth entrepreneur's private information.
  1. These FIs face a problem similar to the one in Williamson (JPE, 1987), according to BGG $\Longrightarrow$ costly state verification.
  2. Costs of monitoring private information source of external finance premium.
  3. Uncollateralized debt more expensive than funding projects with retained earnings, which the entrepreneur lacks.

The BGG Model: FIs Lend to Entrepreneurs

- BGG assume the FI is risk neutral and faces a fixed cost μ when monitoring the jth entrepreneur, where $0 < \mu$.
  1. The FI monitors the return on the market value of the jth entrepreneur's capital stock, $\omega_j R_{K,t+1} Q_t K_{j,t+1}$.
  2. Since ex ante monitoring can fall on any part of the aggregate capital stock in the economy, the relative price of investment to consumption is unaffected by the financial accelerator.

- The FI offers a date t loan that commits the jth entrepreneur to repay at the rate $Z_{j,t+1}$ next period.
  1. If the jth entrepreneur does not default the FI receives $\omega_j R_{K,t+1} Q_t K_{j,t+1} - Z_{j,t+1} B_{j,t+1}$.
  2. Otherwise, the FI recovers $(1 - \mu)\omega_j R_{K,t+1} Q_t K_{j,t+1}$.

The BGG Model: Demand Deposits

- Households deposit cash with FIs for the return $R_{t+1} < R_{K,t+1}$.
- The return R_t represents the FI's opportunity cost of funds.
- BGG argue that the opportunity cost is the riskless rate, R_{t+1}, because FIs diversify the asset side of their balance sheet.

The BGG Model: The Debt Contract in Partial Equilibrium

- BGG assume a FI offers $Z_{j,t+1}$ to the jth entrepreneur borrowing $B_{j,t+1}$ to satisfy the constraint R_t = expected return on the loan.
- Implicit is that the credit market is perfectly competitive.
- Thus, FIs face a zero profit condition that sets the opportunity cost of lending to a weighted average of returns from lending.
- This differs from the problem Williamson studies because the FI also must determine the ω_j at which the jth entrepreneur is monitored.
  1. Label this cutoff $\overline{\omega}_j$, where monitoring occurs when $\omega_j < \overline{\omega}_j$.
  2. The dilemma facing the FI is that it is indifferent to forcing a default or letting the jth entrepreneur repay at $\overline{\omega}_j$.
  3. This is $\overline{\omega}_j = \dfrac{Z_{j,t+1}B_{j,t+1}}{R_{K,t+1}Q_tK_{j,t+1}}$, where by necessity the ex post return on the ex ante value of the capital the jth entrepreneur borrows is greater than the total repayment on that loan.

The BGG Model: The Debt Contract in Partial Equilibrium, cont.

- Given a competitive credit market, holding Q_t, $R_{K,t+1}$, and R_{t+1} fixed, and that the probability of no default, $\omega_j \geq \overline{\omega}_j$, is $1 - \mathcal{F}(\overline{\omega}_j)$, the zero profit condition facing FIs is

$$\begin{aligned} R_{t+1}B_{j,t+1} &= \left[1 - \mathcal{F}(\overline{\omega}_j)\right] Z_{j,t+1}B_{j,t+1} \\ &+ \quad (1-\mu)R_{K,t+1}Q_tK_{j,t+1}\int_0^{\overline{\omega}_j} \omega d\mathcal{F}(\omega). \end{aligned}$$

- Since this condition has two unknowns in one equation, substitute out $Z_{j,t+1}B_{j,t+1}$ using $\overline{\omega}_j R_{K,t+1}Q_tK_{j,t+1}$,

$$\mathcal{R}_t^{-1}\left(1 - \mathcal{K}_{j,t+1}^{-1}\right) = \left[1 - \mathcal{F}(\overline{\omega}_j)\right]\overline{\omega}_j + (1-\mu)\int_0^{\overline{\omega}_j} \omega d\mathcal{F}(\omega).$$

  where $\mathcal{R}_t = \dfrac{R_{K,t+1}}{R_{t+1}}$ and $\mathcal{K}_{j,t+1} = \dfrac{Q_tK_{j,t+1}}{N_{j,t+1}}$.

- With $B_{j,t+1}$ also eliminated from the zero profit condition the FI calculates $\overline{\omega}_j$ given either predetermined variables or variables it takes as given in partial equilibrium.

The BGG Model: The Debt Contract in Partial Equilibrium, cont.

- The zero profit condition

$$\mathcal{R}_t^{-1}\left(1 - \mathcal{K}_{j,t+1}^{-1}\right) = \left[1 - \mathcal{F}(\overline{\omega}_j)\right]\overline{\omega}_j + (1-\mu)\int_0^{\overline{\omega}_j}\omega d\mathcal{F}(\omega).$$

  sets the ex ante external finance premium of the jth entrepreneur equal to the FIs expected return on that loan.

- As the jth entrepreneur's (inverse of the) capital to net worth ratio, $\mathcal{K}_{j,t+1}$, $\longrightarrow 1$, the ex ante external finance premium $\longrightarrow 0$.

- The right side of the zero profit condition is the net expected return to the FI of the loan.
  1. This return nets the ex ante cost, $1-\mu$, of monitoring an entrepreneur whose $\omega_j < \overline{\omega}_j$.
  2. The FI maximizes the return to the loan of the jth entrepreneur that is gross of monitoring costs.

The BGG Model: The Debt Contract in Partial Equilibrium, cont.

- A comparative statics exercise shows that a change in $\overline{\omega}_j$ generates opposing effects on $\mathbf{E}\{\omega_j R_{K,t+1} Q_t K_{j,t+1}\}$, which is the jth entrepreneur's expected return.
- The FI has to trade
  1. the survival probability is non-increasing in $\overline{\omega}_j$ against
  2. a larger $\overline{\omega}_j$ raising the expected return for entrepreneurs that pay off their loans.
- The first effect follows from stretching the upper bound on the integral $-\mu \int_0^{\overline{\omega}_j} \omega d\mathcal{F}(\omega)$ a unit, which is $-\mu \overline{\omega}_j d\mathcal{F}(\overline{\omega}_j) < 0$.
- The second effect follows from evaluating $\dfrac{\partial\left(\left[1 - \mathcal{F}(\overline{\omega}_j)\right]\overline{\omega}_j\right)}{\partial \overline{\omega}_j} \gtreqless 0$.
  1. The result is $-\overline{\omega}_j f(\overline{\omega}_j) + 1 - \mathcal{F}(\overline{\omega}_j) \gtreqless 0$ or
  2. $-\overline{\omega}_j h(\overline{\omega}_j) + 1 > 0$.

The BGG Model: The Debt Contract in Partial Equilibrium, cont.

- The effects of increasing $\overline{\omega}_j$ sum to $1 - \mathcal{F}(\overline{\omega}_j) - \overline{\omega}_j[1+\mu]f(\overline{\omega}_j)$, which equals $[1 - \overline{\omega}_j[1+\mu]h(\overline{\omega}_j)][1 - \mathcal{F}(\overline{\omega}_j)]$.
- Need to show the last expression is a concave function of $\overline{\omega}_j$ (*i.e.*, the second derivative of the zero profit condition is negative).

$$\frac{\partial\Big([1 - \overline{\omega}_j[1+\mu]h(\overline{\omega}_j)]\Big)}{\partial\overline{\omega}_j} = -\left[(1+\mu)h(\overline{\omega}_j) + \overline{\omega}_j\frac{\partial h(\overline{\omega}_j)}{\partial\overline{\omega}_j}\right]$$

$$\Longrightarrow \quad -\left[(1+\mu) + \frac{\overline{\omega}_j}{h(\overline{\omega}_j)}\frac{\partial h(\overline{\omega}_j)}{\partial\overline{\omega}_j}\right] < 0.$$

- The zero profit condition is concave for all $\omega < \overline{\omega}_j$ implying there is one that maximizes the FI's expected return on the loan (*i.e.*, dual to minimizing monitoring costs).
- The comparative static exercise is equivalent to the FI choosing $\overline{\omega}_j$ to maximize $\Big[1 - \mathcal{F}(\overline{\omega}_j)\Big]\overline{\omega}_j + \int_0^{\overline{\omega}_j}\omega d\mathcal{F}(\omega)$ subject to the zero profit condition.

The BGG Model: The Debt Contract in General Equilibrium

- BGG impose an implicit upper bound on R_{t+1} to guarantee $\overline{\omega}_j$ exists in partial equilibrium.
  1. Otherwise, the FI cannot compute $\overline{\omega}_j$, which would
  2. ration the jth entrepreneur out of the credit market.

- Carry this assumption over to BGG's general equilibrium analysis, in which the aggregate shock $\overline{u}$ falls on the expected return to the j entrepreneur, $\overline{u}\omega_j R_{K,t+1} Q_t K_{j,t+1}$.
  1. Risk neutral FIs and entrepreneurs care about expected returns on loans and not higher moments of $\overline{\omega}_j$ and/or $\overline{u}$.
  2. Thus, entrepreneur j is willing to suffer the risk of a small draw of the idiosyncratic shock ω_j or a large aggregate shock to $\overline{u}$.
  3. The FI diversifies the idiosyncratic risk of the ω_js away by lending to a large (countably infinite) number of entrepreneurs.

The BGG Model: The Debt Contract in General Equilibrium, cont.

- Still, how does a FI respond to the aggregate shock $\overline{u}$ that cannot be diversified away?
- BGG show that the aggregate risk of a non-diversifiable shock to $\overline{u}$
  1. forces the repayment commitment of the jth entrepreneur, $Z_{j,t+1}$, to be a state contingent function of $R_{K,t+1}$.
  2. The state dependency of $R_{K,t+1}$ is generated by drawing from the distribution of $\overline{u}$, where $\mathbf{E}\{\overline{u}\} = 1$.
  3. Since $\overline{\omega}_j$ is a function of $R_{K,t+1}$, $Z_{j,t+1}$ is as well.
  4. BGG interpret this state dependent function as a menu of $Z_{j,t+1}$s for every realization of $\overline{u}$.
- BGG claim that ex post $R_{K,t+1}$ and $Z_{j,t+1}$ are negatively correlated.
  1. Changes in $Z_{j,t+1}$ reflect movements in entrepreneurial default probabilities that are tied to variation in $\overline{\omega}_j$.
  2. The latter are inferred from movements in $R_{K,t+1}$.

The BGG Model: The Debt Contract in General Equilibrium, cont.

- Under aggregate risk, a FI's problem is altered in two ways.
- First, the FI sees the jth entrepreneur maximizing the objective

$$\left[1 - \left[1 - \mathcal{F}(\overline{\omega}_j)\right]\overline{\omega}_j - \int_0^{\overline{\omega}_j} \omega f(\omega) d\omega\right] \overline{u}\, \mathcal{R}_{t+1}\mathcal{K}_{j,t+1},$$

where $1 - \mathcal{F}(\overline{\omega}_j) = \int_{\overline{\omega}_j}^{\infty} f(\omega)d\omega = \int_0^{\infty} f(\omega)d\omega - \int_0^{\overline{\omega}_j} f(\omega)d\omega$.

- Next, the FI's zero profit condition is changed to account for $\overline{u}$,

$$\mathcal{K}_{j,t+1} - 1 = \left[\left[1 - \mathcal{F}(\overline{\omega}_j)\right]\overline{\omega}_j + (1-\mu)\int_0^{\overline{\omega}_j} \omega f(\omega) d\omega\right] \overline{u}\, \mathcal{R}_{t+1}\mathcal{K}_{j,t+1}.$$

or

$$1 - \frac{1}{\mathcal{K}_{j,t+1}} = \left[\left[1 - \mathcal{F}(\overline{\omega}_j)\right]\overline{\omega}_j + (1-\mu)\int_0^{\overline{\omega}_j} \omega f(\omega) d\omega\right] \overline{u}\, \mathcal{R}_{t+1}.$$

The BGG Model: The Debt Contract in General Equilibrium, cont.

- With respect to $\overline{\omega}_j$ and $\mathcal{K}_{j,t+1}$, the FONCs are

$$\begin{aligned} -\left[1 - \mathcal{F}(\overline{\omega}_j)\right] + \overline{\omega}_j f(\overline{\omega}_j) - \overline{\omega}_j f(\overline{\omega}_j) & \\ + \lambda_{j,t}\left[1 - \mathcal{F}(\overline{\omega}_j) - \overline{\omega}_j f(\overline{\omega}_j) + (1-\mu)\overline{\omega}_j f(\overline{\omega}_j)\right] &= 0 \end{aligned}$$

and

$$\begin{aligned} \mathbf{E}\Bigg\{\left[1 - \left[1 - \mathcal{F}(\overline{\omega}_j)\right]\overline{\omega}_j - \int_0^{\overline{\omega}_j} \omega f(\omega) d\omega\right]\overline{u}\,\mathcal{R}_{t+1} - \lambda_{j,t} & \\ + \lambda_{j,t}\left[\left[1 - \mathcal{F}(\overline{\omega}_j)\right]\overline{\omega}_j + (1-\mu)\int_0^{\overline{\omega}_j} \omega f(\omega) d\omega\right]\overline{u}\,\mathcal{R}_{t+1}\Bigg\} &= 0. \end{aligned}$$

- $\lambda_{j,t}$ is the Lagrange multiplier attached to the zero profit condition.

The BGG Model: The Debt Contract in General Equilibrium, cont.

- Assuming a no credit rationing equilibrium, the FONC w/r/t $\overline{\omega}_j$ is

$$\lambda_j(\overline{\omega}_j; \mu) = \frac{1 - \mathcal{F}(\overline{\omega}_j)}{1 - \mathcal{F}(\overline{\omega}_j) - \mu\overline{\omega}_j f(\overline{\omega}_j)}.$$

- Thus, the optimal shadow value of an additional unit of return for the FI is independent of the aggregate shock $\overline{u}$ (*i.e.*, the Lagrange multiplier is a function only of $\overline{\omega}_j$, given $\mathcal{F}(\cdot)$, $f(\cdot)$, and μ).

- Also, note that $\lim_{\overline{\omega}_j \longrightarrow 0}\lambda_j(\overline{\omega}_j; \mu) = 1$ and as $\overline{\omega}_j$ goes to a large upper bound, call it $\widehat{\omega}_j$, $\lambda_j(\overline{\omega}_j; \mu) \longrightarrow \infty$.

The BGG Model: The Debt Contract in General Equilibrium, cont.

- Differentiate the optimal λ_j w/r/t $\overline{\omega}_j$, or $\dfrac{\partial \lambda_j(\overline{\omega}_j; \mu)}{\partial \overline{\omega}_j}$, to obtain

$$\frac{\partial \lambda_j}{\partial \overline{\omega}_j} = \mu f(\overline{\omega}_j) \left[\frac{f(\overline{\omega}_j) + \left[1 - \mathcal{F}(\overline{\omega}_j)\right] \left[1 + \dfrac{\overline{\omega}_j}{f(\overline{\omega}_j)} \dfrac{\partial f(\overline{\omega}_j)}{\partial \overline{\omega}_j}\right]}{\left[1 - \mathcal{F}(\overline{\omega}_j) - \mu \overline{\omega}_j f(\overline{\omega}_j)\right]^2} \right].$$

- Since the elasticity of $f(\overline{\omega}_j) \in (-1, 1)$, λ_j is increasing in $\overline{\omega}_j$, or $\dfrac{\partial \lambda_j(\overline{\omega}_j; \mu)}{\partial \overline{\omega}_j} > 0$.

- The potential of a higher default rate is compensated by a higher return having greater value.

- However, λ_j differs across the BGG models with and without $\overline{u}$ because the FIs choice of $\overline{\omega}_j$ is a function of $\mathcal{R}_{t+1}$ and $\mathcal{K}_{j,t+1}$.

- In general equilibrium, shocks to $\overline{u}$ shift $\mathcal{K}_{j,t+1}$.

The BGG Model: The Debt Contract in General Equilibrium, cont.

- The FONC w/r/t $\mathcal{K}_{j,t+1}$ restricts its response to changes in $\mathcal{R}_{t+1}$, shocks to $\overline{u}$, and movements in $\overline{\omega}_j$.
- This suggests that embedded in the FONC w/r/t $\mathcal{K}_{j,t+1}$ is the implicit function $\mathcal{K}_{j,t+1} = \psi(\mathcal{R}_{t+1}; \overline{u}, \overline{\omega}_j)$.
- $\psi(\mathcal{R}_{t+1}; \overline{u}, \overline{\omega}_j)$ is a cost of adjustment function in that a decrease in the external finance premium constrains the aggregate economy's ability to accumulate capital.
- Thus, $\dfrac{\partial \psi(\mathcal{R}_{t+1}; \overline{u}, \overline{\omega}_j)}{\partial \mathcal{R}_{t+1}} > 0$.

The BGG Model: The Debt Contract in General Equilibrium, cont.

- Totally differentiate the FONC w/r/t $\mathcal{K}_{j,t+1}$ by $\mathcal{K}_{j,t+1}$ and $\mathcal{R}_{t+1}$ to show $\dfrac{\partial\psi(\mathcal{R}_{t+1})}{\partial\mathcal{R}_{t+1}} > 0$.
- First, using the zero profit condition and $\mathcal{G}(\overline{\omega}_j) \equiv \int_0^{\overline{\omega}_j} \omega f(\omega)d\omega$, rewrite the FONC w/r/t $\mathcal{K}_{j,t+1}$

$$\mathbf{E}\left\{\left[1 - \mu\mathcal{G}(\overline{\omega}_j)\right]\overline{u}\,\mathcal{R}_{t+1} - \frac{\lambda_j(\overline{\omega}_j)}{\mathcal{K}_{j,t+1}} - 1\right\} = 0.$$

- Subsequent to total differentiation, this condition yields

$$\frac{d\mathcal{K}_{j,t+1}}{d\mathcal{R}_{t+1}} = \frac{\left[1 - \mu\mathcal{G}(\overline{\omega}_j)\right]\overline{u} - \left[\mu\overline{u}\mathcal{R}_{t+1}\overline{\omega}_j f(\overline{\omega}_j) + \dfrac{\lambda_j'(\overline{\omega}_j)}{\mathcal{K}_{j,t+1}}\right]\dfrac{\partial\overline{\omega}_j}{\partial\mathcal{R}_{t+1}}}{\left[\mu\overline{u}\mathcal{R}_{t+1}\overline{\omega}_j f(\overline{\omega}_j) + \dfrac{\lambda_j'(\overline{\omega}_j)}{\mathcal{K}_{j,t+1}}\right]\dfrac{\partial\overline{\omega}_j}{\partial\mathcal{K}_{j,t+1}} + \dfrac{\lambda_j(\overline{\omega}_j)}{\mathcal{K}_{j,t+1}^2}}.$$

The BGG Model: The Debt Contract in General Equilibrium, cont.

- Given $\dfrac{\partial\lambda_j(\overline{\omega}_j)}{\partial\overline{\omega}_j} > 0$, we need to sign $\dfrac{\partial\overline{\omega}_j}{\partial\mathcal{R}_{t+1}}$ and $\dfrac{\partial\overline{\omega}_j}{\partial\mathcal{K}_{j,t+1}}$ to decide if $\dfrac{d\mathcal{K}_{j,t+1}}{d\mathcal{R}_{t+1}} \gtreqless 0$.
- Invoke the implicit function theorem to show the zero profit condition yields

$$\frac{\partial\overline{\omega}_j}{\partial\mathcal{R}_{t+1}} = -\frac{\Big[1-\mathcal{F}(\overline{\omega}_j)\Big]\overline{\omega}_j + (1-\mu)\mathcal{G}(\overline{\omega}_j)}{\Bigg[\Big[1-\mathcal{F}(\overline{\omega}_j)\Big] - \mu\overline{\omega}_j f(\overline{\omega}_j)\Bigg]\mathcal{R}_{t+1}} < 0.$$

- Since the numerator is positive (the survivor probability is greater than the expected probability value at which the jth entrepreneur is monitored), increasing the external finance premium lowers $\overline{\omega}_j$.
- $\overline{\omega}_j$ is falling in $\mathcal{R}_{t+1}$ because the FI can accept greater defaults given project returns are higher.

The BGG Model: The Debt Contract in General Equilibrium, cont.

- Next, compute $\dfrac{\partial \overline{\omega}_j}{\partial \mathcal{K}_{j,t+1}}$.
- The procedure is the same one used to construct $\dfrac{\partial \overline{\omega}_j}{\partial \mathcal{R}_{t+1}} \Longrightarrow$ apply the implicit function theorem to the zero profit condition to obtain

$$\frac{\partial \overline{\omega}_j}{\partial \mathcal{K}_{j,t+1}} = \frac{\mathcal{K}_{j,t+1}^{-2}}{\Big[\big[1 - \mathcal{F}(\overline{\omega}_j)\big] - \mu \overline{\omega}_j f(\overline{\omega}_j)\Big] \overline{u}\, \mathcal{R}_{t+1}} > 0.$$

- As the jth entrepreneur requires more capital, the FI raises the hurdle value of the idiosyncratic shock that the borrower's expected return must clear.
- As the fraction of the FI's balance sheet put at risk by any single entrepreneur increases, the FI demands a greater expected return from the borrower.

The BGG Model: The Debt Contract in General Equilibrium, cont.

- Remember that

$$\frac{d\mathcal{K}_{j,t+1}}{d\mathcal{R}_{t+1}} = \frac{\Big[1 - \mu G(\overline{\omega}_j)\Big]\overline{u} - \left[\mu\overline{u}\mathcal{R}_{t+1}\overline{\omega}_j f(\overline{\omega}_j) + \frac{\lambda_j'(\overline{\omega}_j)}{\mathcal{K}_{j,t+1}}\right]\frac{\partial\overline{\omega}_j}{\partial\mathcal{R}_{t+1}}}{\left[\mu\overline{u}\mathcal{R}_{t+1}\overline{\omega}_j f(\overline{\omega}_j) + \frac{\lambda_j'(\overline{\omega}_j)}{\mathcal{K}_{j,t+1}}\right]\frac{\partial\overline{\omega}_j}{\partial\mathcal{K}_{j,t+1}} + \frac{\lambda_j(\overline{\omega}_j)}{\mathcal{K}_{j,t+1}^2}}.$$

- Since $\frac{\partial\overline{\omega}_j}{\partial\mathcal{R}_{t+1}} < 0$ and $\frac{\partial\overline{\omega}_j}{\partial\mathcal{K}_{j,t+1}} > 0$, $\frac{d\mathcal{K}_{j,t+1}}{d\mathcal{R}_{t+1}} \equiv \frac{\partial\psi(\mathcal{R}_{t+1})}{\partial\mathcal{R}_{t+1}} > 0$.

- Relative to net worth, the jth entrepreneur expands the value of its project only when the external finance premium increases.

- A FI needs a larger external finance premium to compensate for the risk induced by offering a larger loan to the jth entrepreneur.

The BGG Model: The Entrepreneur's Problem with Aggregate Risk

- Entrepreneurs aim to maximize the expected return on their successful projects, which is net of those with $\omega_j \leq \overline{\omega}_j$,

$$\mathbf{E}\left\{\left[\int_{\overline{\omega}_j}^{\infty} \omega f(\omega) d\omega - \left[1 - \mathcal{F}(\overline{\omega}_j)\right] \overline{\omega}_j\right] \overline{u} R_{K,t+1} \middle| \overline{u}, R_{K,t+1}\right\} Q_t K_{j,t+1}.$$

- Unlike FIs, entrepreneurs' demand capital (or credit) is conditional on the expected return to capital.
- The demand for $K_{j,t+1}$ depends on $\overline{u}$ and $R_{K,t+1}$ because $\overline{\omega}_j$ is.
- Entrepreneurs also condition on expected defaults because their risk neutrality means they absorb losses associated with badly performing projects, given FIs are perfectly competitive.

The BGG Model: The Entrepreneur's Problem with . . . , cont.

- This is made clear by substituting for $\Big[1 - \mathcal{F}(\overline{\omega}_j)\Big]\overline{\omega}_j$ using the FI zero profit condition

$$\left[\frac{Q_t K_{j,t+1} - N_{j,t+1}}{Q_t K_{j,t+1}}\right]\frac{R_{t+1}}{\overline{u}R_{K,t+1}} - (1-\mu)\int_0^{\overline{\omega}_j}\omega f(\omega)d\omega = \Big[1 - \mathcal{F}(\overline{\omega}_j)\Big]\overline{\omega}_j.$$

- The result is

$$\mathbf{E}\left\{\Big[\int_{\overline{\omega}}^{\infty}(1-\mu)\int_0^{\overline{\omega}_j}\Big]\omega f(\omega)d\omega\, R_{K,t+1}\right\} - R_{t+1}\big(Q_t K_{j,t+1} - N_{j,t+1}\big) =$$

$$\mathbf{E}\left\{\Big[1 - \mu\int_0^{\overline{\omega}_j}\omega f(\omega)d\omega\Big]\overline{u}\epsilon_{R,K,t+1}\right\}\mathbf{E}\Big\{R_{K,t+1}\Big\} - R_{t+1}\big(Q_t K_{j,t+1} - N_{j,t+1}\big),$$

where $\epsilon_{R,K,t+1}$ is the Euler error $R_{K,t+1}\Big/\mathbf{E}\Big\{R_{K,t+1}\Big\}$.

- Thus, the jth entrepreneur's return is net of expected losses generated by default and loan repayment.

The BGG Model: The Entrepreneur's Problem with . . . , cont.

- The jth entrepreneur maximizes its net expected return by choosing $K_{j,t+1}$ and from a menu of $\overline{\omega}_j$s offered by a FI.
  1. The FI solves the jth entrepreneur's problem to construct the menus of $\overline{\omega}_j$.
  2. Similarly, the jth entrepreneur is constrained by FI's zero profit condition.
  3. $\Longrightarrow$ the FI has to recover its opportunity cost of funds.

- The zero profit condition constraint implies that $R_{K,t+1} > R_{t+1}$ for all dates t the jth entrepreneur to expect
  1. its project to be profitable net of expected default costs while
  2. agreeing to the repayment menu of $Z_{j,t+1}$s the FI offers.

The BGG Model: General Equilibrium, cont.

- Having solved the FI's and the jth entrepreneur's problems, we see
  1. the former is offering the latter a menu of $\overline{\omega}_j$ ($\Longrightarrow Z_{j,t+1}$s), given Q_t, $R_{K,t+1}$, and $K_{j,t+1}$ (or $\mathcal{K}_{j,t+1}$), while
  2. the latter demands $K_{j,t+1}$ given $\mathbf{E}\{R_{K,t+1}\}$, Q_t, $N_{j,t+1}$ and R_{t+1}.
  3. $\Longrightarrow$ the FI faces a zero profit condition, which covers $R_{t+1}B_{t+1}$.

- The general equilibrium implication of solving these two optimization problem is $\mathcal{K}_{j,t+1} = \psi(\mathcal{R}_{t+1})$.

- However, the argument of the cost of adjustment function $\psi(\cdot)$ is $\mathbf{E}\{\mathcal{R}_{t+1}\}$ ($= \mathbf{E}\{R_{K,t+1}/R_{t+1}\}$) to reflect that entrepreneurs decide on their $K_{j,t+1}$ prior to the realization of $R_{K,t+1}$.

- Thus, $Q_tK_{j,t+1} = \psi(\mathbf{E}\{\mathcal{R}_{t+1}\})N_{j,t+1}$, where
  1. $\psi(1) = 1$, $\Longrightarrow$ projects are financed with internal funds forcing the expected external finance premium to disappear and
  2. $\psi'(\cdot) > 0 \Longrightarrow$ an increase in the expected external finance premium lowers the probability of default allowing entrepreneurs to borrow more and scale up their projects.

The BGG Model: General Equilibrium, cont.

- BGG exploit the inverse function theorem to produce,
  1. $\mathbf{E}\{R_{K,t+1}\} = \mathcal{R}\left(\frac{N_{j,t+1}}{Q_t K_{j,t+1}}\right) R_{t+1}$, where $\mathcal{R}(\cdot) = \psi^{-1}(\cdot)$, but
  2. nothing is said either about bounds on which $\psi'(\cdot)$ exists implying that $\mathcal{R}(\cdot) = \psi^{-1}(\cdot)$ is true or whether there is a contraction mapping (or fixed point) establishing the result.
  3. Nonetheless, the assumption is $\mathcal{R}'(\cdot) > 0$.
- Since the expected return to capital measures the discounted expected flow of income to projects,
  1. $\mathbf{E}\{R_{K,t+1}\}$ equals, what BGG call, the entrepreneurial "marginal cost of external finance function," $\mathcal{R}(\cdot)$,
  2. which is centered on the opportunity cost of funds, R_t, for FIs.
  3. This relationship represents equilibrium in the credit market.
- Thus, $\mathbf{E}\{R_{K,t+1}\}$ is a function of the inverse of the project's scale to the jth entrepreneur's net worth $\Longrightarrow$ the jth entrepreneur's own share of the project.

The BGG Model: General Equilibrium and Aggregation

- BGG give entrepreneurs the CRS technology, $A_t K_{j,t}^{\alpha} L_{j,t}^{1-\alpha}$, for all j, to produce the intermediate good $Y_{j,t}$, where $L_{j,t}$ is labor demand of the jth entrepreneur.
  1. CRS primitives induce a map that equates entrepreneurial demand for capital with a multiple of net worth, which
  2. allows BGG to average across the j entrepreneurs to create the aggregate intermediate production technology, $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$.
  3. Thus, avoid having the distribution of net worth across the j entrepreneurs as a state variable, which
  4. eliminates the higher order moments of this distribution as drivers of the state dynamics of the BGG-NKDSGE model.
  5. As a result, idiosyncratic shocks are washed out of this DSGE model's aggregate fluctuations.

The BGG Model: General Equilibrium and Aggregate Investment

- BGG assume the law of motion of the aggregate capital stock is

$$K_{t+1} = \left[1 + \Phi\left(\frac{I_t}{K_t}\right) - \delta\right] K_t, \;\; \delta \in \left(0, 1\right),$$

  where $\Phi(0) = 0$, $\Phi'(\cdot) > 0$, and $\Phi''(\cdot) < 0$.

- Note that investment is not the flow of newly produced final goods into the stock of capital.
- Instead, $\Phi(\cdot)$ is the function generating new capital.
- This is a cost of adjustment function in the investment-capital ratio.
  1. When this function moves, the price of capital, Q_t, fluctuates.
  2. BGG recognize this relationship with $Q_t = \left[\Phi'\left(\frac{I_t}{K_t}\right)\right]^{-1}$.
  3. Thus, Q_t moves inversely with the I_t increasing relative to K_t.
  4. Normalize the steady state price of capital, Q^*, to one.

The BGG Model: Aggregate Return to Capital in General Equilibrium

- The aggregate demand for capital is not generated by inverting the marginal product of capital, $\text{MP}_{K,t} = \alpha Y_t / K_t$.
- A reason is there are two (intermediate and final) goods.
  1. Let X_t^{-1} be the relative price of intermediate to final goods.
  2. Thus, entrepreneurs pay $\frac{\alpha Y_t}{X_t K_t}$ to rent capital.
  3. The ex post value of unit of capital tomorrow is $(1 - \delta)Q_{t+1}$.
  4. Since the relative price of capital to final goods is Q_t^{-1}, it is the opportunity cost the aggregate economy faces when accumulating a unit of capital.
- The expected return to capital equals the opportunity cost of accumulating a unit of capital multiplied by the sum of the expected rental rate of capital plus the expected value of a unit of capital.

$$\mathbf{E}_t\left\{R_{K,t+1}\right\} = \frac{1}{Q_t}\mathbf{E}_t\left\{\frac{\alpha Y_{t+1}}{X_{t+1}K_{t+1}} + (1 - \delta)Q_{t+1}\right\}.$$

The BGG Model: Aggregate Capital Demand in General Equilibrium

- Substitute for Q_t using $\Phi'\left[\left(\frac{I_t}{K_t}\right)\right]^{-1}$ to obtain

$$\mathbf{E}_t\left\{R_{K,t+1}\right\} = \frac{1}{\left[\Phi'\left(\frac{I_t}{K_t}\right)\right]^{-1}}\mathbf{E}_t\left\{\frac{\alpha Y_{t+1}}{X_{t+1}K_{t+1}} + (1-\delta)\left[\Phi'\left(\frac{I_{t+1}}{K_{t+1}}\right)\right]^{-1}\right\}.$$

- $\Longrightarrow$ Implicit demand function for K_{t+1} conditional on K_t.
  1. The demand for expected capital also depends on I_t
  2. and expected investment at date $t+1$.
- The supply of capital is restricted by the function

$$\mathbf{E}\{R_{K,t+1}\} = \mathcal{R}\left(\frac{N_{t+1}}{Q_tK_{t+1}}\right)R_{t+1}.$$

- Variation in the aggregate supply function of capital relies on the changes in the state of aggregate financial conditions, $\frac{N_{t+1}}{Q_tK_{t+1}}$.

The BGG Model: Aggregate Labor Demand in General Equilibrium

- BGG assume that entrepreneurs supply their labor, L_e, inelastically.
- Entrepreneurial labor is mixed with household labor, $L_{h,t}$, which is in elastic supply, in a CRS technology $L_{h,t}^{\vartheta} L_e^{1-\vartheta}$ to generate aggregate labor input L_t.
  1. CRS labor technology require entrepreneurs to work on projects other than their own.
  2. $\Longrightarrow$ entrepreneurs have special skills needed by a wide range of projects.
  3. The technical issue is that $L_{e,j}$ must differ across projects to obey the scaling implied by CRS production technologies.

The BGG Model: Aggregate Equity Dynamics in General Equilibrium

- BGG define entrepreneurial equity as $V_{j,t}$, which the jth entrepreneur accumulates by operating successful projects.

- From date $t-1$ to t, aggregate entrepreneurial equity evolves as

$$V_t = \left[\left(R_{K,t} - R_t \right) - \mu R_{K,t} \int_0^{\overline{\omega}} \omega f(\omega) d\omega \right] Q_{t-1} K_t + R_t N_t.$$

1. The first bit is the excess return on the project.
2. The next is the expected loss on failed projects.
3. The final piece is the (safe) return on net worth.
4. The entrepreneurial sector consumes a fixed fraction of equity, $C_{e,t} = (1-\gamma)V_t$, that is the share tied to projects in default.

- A fraction, γ, of $V_{j,t}$ plus the jth entrepreneur's labor income, $W_{e,j}$, equals $N_{j,t+1}$ and in the aggregate $N_{t+1} = \gamma V_t + W_e$.

The BGG Model: The Aggregate Net Worth Law of Motion

- Since $\ln \varepsilon_{R,K,t+1} \approx \varepsilon_{R,K,t+1} = R_{K,t+1} - \mathbf{E}_t\left\{R_{K,t+1}\right\}$,

$$V_t - \mathbf{E}_{t-1}\left\{V_t\right\} = \left[\varepsilon_{R,K,t} - \mu\varepsilon_{R,K,\overline{\omega},t}\right]Q_{t-1}K_t,$$

where $\varepsilon_{R,K,\overline{\omega},t} = R_{K,t}\int_0^{\overline{\omega}}\omega f(\omega)d\omega - \mathbf{E}_t\left\{R_{K,t}\int_0^{\overline{\omega}}\omega f(\omega)d\omega\right\}$.

- We want to know the response of V_t to $\varepsilon_{R,K,t}$, which is
  1. $\dfrac{\partial V_t}{\partial \varepsilon_{R,K,t}} = Q_{t-1}K_t$ or as the elasticity $\dfrac{\varepsilon_{R,K,t}}{V_t}\dfrac{\partial V_t}{\partial \varepsilon_{R,K,t}}$.
  2. $\dfrac{\varepsilon_{R,K,t}}{V_t}\dfrac{\partial V_t}{\partial \varepsilon_{R,K,t}} = \left(R_{K,t} - \mathbf{E}_{t-1}\left\{R_{K,t}\right\}\right)\dfrac{Q_{t-1}K_t}{V_t} \in (-1,\ 1)$.

- Innovations to the return to capital can lead to an increase or a drop in entrepreneurial equity.

- The less equity entrepreneurs have invested in the project the greater is the elasticity (in absolute value) of equity to an unexpected change in the return to capital.

The BGG Model: The Aggregate Net Worth Law of Motion, cont.

- BGG assert $N_{t+1} = \gamma V_t + W_e \Longrightarrow$ changes in the value of total equity across all firms drive fluctuations in aggregate net worth.
- Substitute for V_t to find the equilibrium law of motion of aggregate entrepreneurial net worth

$$N_{t+1} = \gamma\Bigg(\Big[\Big(R_{K,t} - R_t\Big) - \mu R_{K,t}\int_0^{\overline{\omega}} \omega f(\omega)d\omega\Big]Q_{t-1}K_t + R_t N_t\Bigg) + (1-\alpha)(1-\vartheta)\frac{Y_t}{L_t},$$

where the marginal product of entrepreneurial labor in the second line suggests that there is time-variation in L_e.

- Since movements in N_{t+1} generate changes in $\mathbf{E}_t\{R_{K,t+1}\}$, the equilibrium law of motion of N_{t+1} and the capital supply function $\mathcal{R}\left(\frac{N_{t+1}}{Q_t K_{t+1}}\right)$ describes the financial accelerator of the BGG model.

Summary: FIs and the Financial Accelerator

- Partial equilibrium models of financial crises have a hard time being cast as general equilibrium stories.
- Whether costly state verification or a principal-agent problem create the potential for firm default in partial equilibrium, does not necessarily produce financial crises in DSGE models.
- Lenders absorb losses tied to firm defaults without becoming insolvent in Williamson's and BGG's models except when defaults are produced by large negative aggregate shocks.
- Williamson relies on diversification for this result $\Longrightarrow$ assume a large mass of relatively riskless borrowers exist.
- This result rests on two assumptions in a BGG model.
  1. A borrower's probability of not defaulting has bounded support and borrowers' net worth and equity are equivalent in the aggregate.
  2. $\Longrightarrow$ In net, entrepreneurs' aggregate balance sheet has positive value.
  3. Whether a BGG model fits actual data better, say, compared with a canonical medium scale NK model remains an open question; see Brzoza-Brzezina and Kolasa (2013).

Kiyotaki and Moore (1997)

- Kiyotaki and Moore (KM) innovate by letting borrowers offer their durable goods as collateral to lenders.
- Firms do not necessarily have to borrow to purchase or rent durable goods, but these goods must be valued in production.
- Borrowers need credit to purchase other factor inputs to produce.
  1. The value of the durable good determines the shadow price of the collateral constraint.
  2. Expectations about future output are driven by shifts in credit constraints today, which feedback onto the current value of the durable good.
  3. Implicit in KM models is that the value of the durable good to lenders is less than it is to borrowers.
- Thus, borrowing constraints are endogenous in KM models.

A KM Example: A Model of Agricultural Production

- The KM model needs at least two goods.
- Producers own land $\Longrightarrow$ farm land.
  1. Land is the economy's durable good,
  2. is not consumed, and
  3. is in fixed supply.
- The nondurable good is consumed $\Longrightarrow$ grain.
- Assume there are two types of firms which can combine land and the nondurable good to produce more of the latter.
  1. One type of firm is more patient, carries relatively less debt, and or has access to a safe technology.
  2. The remaining firms are less patient or have large debt to land value ratios $\Longrightarrow$ leverage ratio.
- Prices are perfectly flexible and there is no private information.

A KM Example: Propagation

- Let leveraged firms experience a bad productivity shock at date t.
- The bad shock results in less date t output implying land is less valuable today.
- Declining land values tighten collateral constraints $\Longrightarrow$ the net worth of constrained firms fall.
- Leveraged firms bump up against these constraints when borrowing to purchase the nondurable good for date $t+1$ production.
- Date $t+1$ output is lower because there is less nondurable good available for production.
- Repeating this process creates, what is called “knock-on effects.”
  1. The knock-on effect is a propagation mechanism.
  2. A shock at date t has long lasting effects.
  3. There is also a within period mechanism that drives volatility.

Market Clearing and the KM Static Multiplier

- At each date t along the equilibrium path, either the market for land or for credit clears ($\Longrightarrow$ Walras' law?).
- Unconstrained firms take the demand side of the market when the analysis focuses on land.
- Consider a KM economy in the bad productivity state of the world.
  1. Constrained firms need more resources to payoff their debt $\Longrightarrow$ their output is falling.
  2. The only source of revenue available comes from selling land.
  3. More land is brought to market by constrained firms.
  4. This pushes down the opportunity cost of land.
  5. $\Longrightarrow$ unconstrained firms increase their demand for land.
- An *intra*-temporal adjustment mechanism clears the market for land within any date t $\Longrightarrow$ creates volatility but not persistence.

Market Clearing and the KM Intertemporal Multiplier

- For unconstrained firms, the opportunity cost of land is the date t land price net of the expected PDV of the flow of future income (*i.e.*, value of output) garnered by owning land.
- Only if the expected PDV of this future income flow tied drops more than the current land price will the opportunity cost of land fall.
- Land owners anticipate that the drop in the expected PDV of their future flow of income will generate lower land prices in the future.
- More land is expected to be shed by constrained firms in the future.
  1. Collateral constraints may be tighter in the future.
  2. The date t price of land can decline further leading constrained firms to sale more land today.
  3. $\Longrightarrow$ In a bad state of the world, the durable good is sold by borrowers at a "fire sale" price.
- This process is repeated until a new equilibrium is established at a lower price of land

KM Intratemporal and Intertemporal Multipliers

- The KM model generates greater intratemporal volatility
  1. by tying the value of a durable good that cannot be produced to productivity shocks and
  2. by equating the borrowing constraints imposed on low net worth firms with the value of the land they own, which
  3. generates movements in the net worth of firms.

- The propagation mechanism rests on several elements.
  1. Changes in the expected PDV of the flow of future income from owning land drives intertemporal choices about land ownership by constrained firms.
  2. Constrained firms may become more so and some unconstrained firms may find themselves facing collateral constraints as the value of land falls.
  3. Unconstrained firms anticipate that in the future this process will be repeated.

Krishnamurthy (JET, 2003)

- KM conclude with

  The interaction between asset markets and credit markets that we have highlighted in this paper will be even richer if both sides of the credit market are affected by changes in the price of their collateralized assets. (p. 244)

- Krishnamurthy addresses this issue by adding agents to the model who offer insurance or options to constrained firms.
  1. The option pays off in the bad state of the world just when constrained firms need grain.
  2. In the bad state of the world, the "fire sale" price of land is low because the constrained firm has to sell its durable good to obtain grain to consume.
  3. The option hedges the risk a constrained firms faces when confronted by a fire sell.

Collateral Constraints and Hedging

- Begs the question of the "completeness" of options markets.
  1. Producers are not constrained if options markets are complete.
  2. $\Longrightarrow$ Producers buy insurance to equalize the marginal utility of consumption across every state of the world.

- This equality breaks down when insurers face collateral constraints $\Longrightarrow$ the KM results about persistence and volatility are restored.

- The question is how tight are the collateral constraints on insurers.
  1. Given insurers face collateral constraints, there is excess demand for options by constrained producers, but
  2. the extent of this gap is determined by the value of the collateral posted by insurers.

Krishnamurthy's KM Model: Set Up

- The economy lasts three periods and consists of
  1. a durable good (*i.e.*, land) in fixed supply = $\overline{K}$,
  2. a nondurable consumption good called grain,
  3. and unit measures of B(anks) and F(armers) firms.
- Bs and Fs have preferences over the uncertain consumption streams of their three period lives $\mathcal{U} = \mathbf{E}\{c_0 + c_1 + c_2\}$.
- At dates 0, 1, and 2, consumption equals the "dividends" gained from operating either a B or F firm.
- The Bs own $\overline{K}$ and receive large endowments of grain at the start of dates 0, 1, and 2.
- F firms receive $w_{F,0}$ at the start of date 0, which is smaller than the endowment B receives at that time $\Longrightarrow$ the Fs borrow from the Bs.

Krishnamurthy's KM Model: Set Up, cont.

- F firms own a technology $f(\cdot)$, which combines land and grain in fixed proportions (*i.e.*, a Leontief production function).
  1. In date 0, K_0 units are mixed with $G_0 = \alpha K_0$ units of grain $\Longrightarrow zf(\min[K_0, G_0/\alpha])$ yields $y_{F,1}$ at the start of date 1.
  2. z is a technology shock common to F firms realized as z_L or z_H,
  3. where $z_L < z_H$, $\Pr(z = z_L) = \pi$, and $\Pr(z = z_H) = 1 - \pi$.
  4. At date 1, K_1 units are mixed with $G_1 = \alpha K_1$ units of grain $\Longrightarrow f(\min[K_1, G_1/\alpha])$ yields $y_{F,2}$ at the start of date 2.
  5. Date 0 production is stochastic, but production is not in date 1.

Krishnamurthy's KM Model: Set Up, cont.

- B firms own a non-stochastic technology $b(\min[K, G/\alpha])$, which in dates 0 and 1 produces $y_{B,1}$ and $y_{B,2}$.
- Assume the technology of F and B firms has decreasing returns to scale (DRS): $f(x) = b(x) \equiv x(\mathcal{A} - x)$.
- Impose the restrictions $2\overline{K} > \mathcal{A} - \alpha$ and $\mathcal{A} - \alpha > \overline{K}$ on the production technology
    1. to guarantee a strictly positive price of land
    2. but there are two equilibria in which the price of land > 0.
    3. $\Longrightarrow$ We will see the restrictions forces a linear supply function to cross a concave demand function twice.

Krishnamurthy's KM Model: A F Firm's Constraints sans Hedging

- A F firm borrows $\mathcal{D}$ to fund $K_{F,0}$ units of land for date 0 production,
  1. but faces the collateral constraint $\mathcal{D} \leq q_1(z_H)K_{F,0}$,
  2. where $q_1(\cdot)$ is the price of land (per unit of grain) at date 1.
  3. The constraint prices $K_{F,0}$ at $q_1(z_H)$ to collateralize $\mathcal{D}$ fully.
  4. $\Longrightarrow$ The maximum that F can repay.

- The date 0 budget constraint of a F firm is $\big(\alpha + q_0\big)K_{F,0} \leq \mathcal{D} + w_{F,0}$.

- A F firm has available $\mathcal{W}_{F,1}(z) = q_1K_{F,0} + zf(K_{F,0}) - \mathcal{D}$ units of grain at date 1, where $\mathcal{W}_{F,1}(z)$ is a F firm's wealth at that date
  $\Longrightarrow$ the date 1 budget constraint is $\big(\alpha + q_1\big)K_{F,1} \leq \mathcal{W}_{F,1}$.

- Since $q_2 = 0$, there cannot be debt at date 2 $\Longrightarrow$ $\mathcal{W}_{F,2}(z) = f(K_{F,1})$, where the F firm buys $K_{F,1}$.

- F firms consume all date 1 production during date 2.

Krishnamurthy's KM Model: A F Firm's Date 1 Problem sans Hedging

- The F firm's problem at date 1 is to choose $K_{F,1}(z)$ to maximize $f\left(K_{F,1}(z)\right) + \mathcal{W}_{F,1}(z) - \left[\alpha + q_1(z)\right] K_{F,1}(z)$, subject to

$$\left[\alpha + q_1(z)\right] K_{F,1}(z) \leq \mathcal{W}_{F,1}(z).$$

- Conditional on whether the F firm is collateral constrained, there are two solutions.
  1. An unconstrained firm's demand for $K_{F,1}(z)$ is $f'^{-1}\left(\alpha + q_1(z)\right)$, $\Longrightarrow$ the F firm's date 1 budget constraint holds with equality.
  2. At wealth $\overline{\mathcal{W}}_{F,1}(z) > \mathcal{W}_{F,1}(z)$, the constraint binds strictly, $\Longrightarrow \left[\alpha + q_1(z)\right] K_{F,1}(z) - \mathcal{W}_{F,1}(z) > 0$.

- Thus, the F firms demand function for land is

$$K_{F,1}(z) = f'^{-1}\left(\alpha + q_1(z)\right) - \frac{1}{\alpha + q_1(z)} \text{Max}\left[\overline{\mathcal{W}}_{F,1}(z) - \mathcal{W}_{F,1}(z),\, 0\right].$$

Krishnamurthy's KM Model: A F Firm's Date 0 Problem sans Hedging

- A F firm maximizes the expected value of its wealth at date 0, $\mathcal{J}\Big(\mathcal{W}_{F,1}(z)\Big)$, s.t. $\Big[\alpha + q_0(z)\Big]K_{F,0}(z) \leq \mathcal{D} + w_{F,0}$ and $\mathcal{D} \leq q_1(z)K_{F,0}$.

- Date 1 wealth is valued according to the function

$$\mathcal{J}\Big(\mathcal{W}_{F,1}(z)\Big) = \mathcal{W}_{F,1}(z) - \overline{\mathcal{W}}_{F,1}(z) + f\left(\frac{\overline{\mathcal{W}}_{F,1}(z)}{\alpha + q_1(z)}\right)$$
$$- \ \text{Max}\left[\mathcal{W}_{F,1}(z) - \overline{\mathcal{W}}_{F,1}(z) + f\left(\frac{\overline{\mathcal{W}}_{F,1}(z)}{\alpha + q_1(z)}\right) - f\left(\frac{\mathcal{W}_{F,1}(z)}{\alpha + q_1(z)}\right), 0\right].$$

- For $\mathcal{W}_{F,1}(z) < \overline{\mathcal{W}}_{F,1}(z)$, $\dfrac{\partial\mathcal{J}\Big(\mathcal{W}_{F,1}(z)\Big)}{\partial\mathcal{W}_{F,1}(z)} = 1 + f'\Big(K_{F,1}(z)\Big)$ and, given $f(x) = x(\mathcal{A} - x)$, $\dfrac{\partial^2\mathcal{J}\Big(\mathcal{W}_{F,1}(z)\Big)}{\partial\mathcal{W}_{F,1}(z)^2} < 0, \Longrightarrow \mathcal{J}\Big(\mathcal{W}_{F,1}(z)\Big)$ is strictly concave.

- $\mathcal{J}\Big(\mathcal{W}_{F,1}(z)\Big)$ is linear when $\mathcal{W}_{F,1}(z) \geq \overline{\mathcal{W}}_{F,1}(z)$, because the constraint moves one for one with the value of the F firm's wealth.

Krishnamurthy's KM Model: A B Firm's Problem sans Hedging

- There are no collateral constraints on the risk neutral B firms.
  1. $\Longrightarrow b'(K_{B,0}) = \alpha + q_0 - \mathbf{E}\Big\{q_1(z)\Big\}$ and $b'\Big(K_{B,1}(z)\Big) = \alpha + q_1(z)$.
  2. Marginal products of capital equal the marginal cost of buying an additional unit of land and planting an extra kernel of grain.
- Risk neutrality and no discounting also suggests that B firms charge a zero (net) rate of interest on loans to F firms.
- This is optimal for B firms only if they are not collateral constrained.

Krishnamurthy's KM Model: Equilibrium sans Hedging

- The land, grain, and credit market must clear for an equilibrium at dates 0, 1, and 2.
- Given land is not reproducible, equilibrium requires
  1. Date 0: $\overline{K} = K_{B,0} + K_{F,0}$.
  2. Date 1: $\overline{K} = K_{B,1}(z) + K_{F,1}(z)$.
- The credit market clears at a gross interest rate of one in dates 0 and 1 $\Longrightarrow$ the grain market clears by Walras' Law.
- The equilibrium supply function of land is found by
  1. substituting $K_{B,1}(z) = \overline{K} - K_{F,1}(z)$ into $b'\left(K_{B,1}(z)\right)$ to show
  2. $q_1(z) = b'\left(\overline{K} - K_{F,1}(z)\right) - \alpha = \mathcal{A} - \alpha - 2\left[\overline{K} - K_{F,1}(z)\right]$.
  3. See Figure 1.

Krishnamurthy's KM Model: Proposition 1

- Suppose $\mathcal{W}_{F,1}(z) < \overline{\mathcal{W}}_{F,1}(z) \Longrightarrow$ the F firm is collateral constrained.
- An increase $\mathcal{W}_{F,1}(z)$ generates a linear increase in $K_{F,1}(z)$; see

$$K_{F,1}(z) = f'^{-1}\Big(\alpha + q_1(z)\Big) - \frac{1}{\alpha + q_1(z)}\text{Max}\Big[\overline{\mathcal{W}}_{F,1}(z) - \mathcal{W}_{F,1}(z),\, 0\Big].$$

- Remember that in equilibrium, $q_1(z) = \mathcal{A} - \alpha - 2\Big[\overline{K} - K_{F,1}(z)\Big]$ $\Longrightarrow$ an increase $\mathcal{W}_{F,1}(z)$ has a similar impact on $q_1(z)$.
- Also, remember that $\mathcal{W}_{F,1}(z) = q_1(z)K_{F,0} + zf(K_{F,0}) - \mathcal{D}$.
  1. Suppose $z = z_L \Longrightarrow z_L f(K_{F,0})$ and $q_1(z_L)$ are low.
  2. Ignore changes in $q_1(z_L)$ on $\mathcal{W}_{F,1}(z_L)$ for the moment.
  3. Date 1 output and wealth fall $\Longrightarrow$ KM's volatility effect.
  4. With wealth and $q_1(z_L)$ lower, the collateral constraint, $\Big[\alpha + q_1(z)\Big]K_{F,1}(z) - \mathcal{W}_{F,1}(z) > 0$, is tighter (the gap between $\overline{\mathcal{W}}_{F,1}(z_L)$ and $\mathcal{W}_{F,1}(z_L)$ widens) moving from date 1 to date 2 $\Longrightarrow$ KM's persistence effect.

Krishnamurthy's KM Model: Demand for the Hedge

- Remember the firms value function for date 1 wealth is

$$\mathcal{J}\Big(\mathcal{W}_{F,1}(z)\Big) = \mathcal{W}_{F,1}(z) - \overline{\mathcal{W}}_{F,1}(z) + f\left(\frac{\overline{\mathcal{W}}_{F,1}(z)}{\alpha + q_1(z)}\right)$$

$$- \text{Max}\left[\mathcal{W}_{F,1}(z) - \overline{\mathcal{W}}_{F,1}(z) + f\left(\frac{\overline{\mathcal{W}}_{F,1}(z)}{\alpha + q_1(z)}\right) - f\left(\frac{\mathcal{W}_{F,1}(z)}{\alpha + q_1(z)}\right), 0\right].$$

- Suppose $\mathcal{W}_{F,1}(z) < \overline{\mathcal{W}}_{F,1}(z) \Longrightarrow$ the collateral constrained F firm is on the concave part of its value function.
  1. The F firm is risk averse in this case (because $f(\cdot)$ is concave).
  2. $\Longrightarrow$ desire to insure against z_L state of the world in which $\mathcal{D}$ cannot be paid back.
  3. Insurance restores linearity and risk neutrality to the economy by achieving $\mathcal{W}_{F,1}(z_L) = \mathcal{W}_{F,1}(z_H)$.
  4. $\Longrightarrow$ eliminate concavity of F firm's value function; see Figure 2.

Krishnamurthy's KM Model: The Hedge

- Add a complete set of securities contingent on the realization of the state of the world, $z = [z_L, z_H]$, that the B firms offer the F firms.
- Let $\theta_F(z)$ denote the options a F firm buys at date 0.
  1. An option is a claim on a specified amount of grain given the realization of z.
  2. When $\mathcal{W}_{F,1}(z_L) < \overline{\mathcal{W}}_{F,1}(z_L)$ and $\mathcal{W}_{F,1}(z_H) < \overline{\mathcal{W}}_{F,1}(z_H)$, there is also a limit to the amount of claims a collateral constrained F firm can purchase.
- The options market alters a F firm's date 1 wealth and collateral constraint to
  1. $\mathcal{W}_{F,1}(z) = q_1(z)K_{F,0} + zf(K_{F,0}) - \mathcal{D} + \theta_F(z)$ and
  2. $\theta_F(z) \geq \mathcal{D} - q_1(z)K_{F,0}$.
  3. The F firm uses the option to "loosen" its collateral constraint, especially in the z_L state of the world.

Krishnamurthy's KM Model: A F Firm's Date 0 Problem with Hedging

- Since the F firm is hedging its output, the loan is state contingent.
  1. $\Longrightarrow \mathcal{D}(z) = q_1(z)K_{F,0} \Longrightarrow \theta_F(z) \geq 0 \Longrightarrow$ a short sell constraint.
  2. This short sell constraint forces the value of the option to be at least that of the state contingent value of firm F's debt net of the state contingent market value of land.
  3. Rules out bets that the F firm will fail.
- At date 0, the F firm solves $\text{Max}_{\{K_{F,0},\,\theta_F(z)\}}\mathbf{E}\Big\{\mathcal{J}\Big(\mathcal{W}_{F,1}(z)\Big)\Big|\,z\Big\}$, s.t.
  1. $\mathcal{W}_{F,1}(z) = zf(K_{F,0}) + \theta_F(z)$.
  2. $\sum\phi(z)\theta_F(z) + \Big(\alpha + q_0\Big)K_{F,0} \leq \sum\phi(z)q_1(z)K_{F,0} + w_{F,0}$, where $\phi(z)$ is the price of an option per unit of grain.
  3. $\theta_F(z) \geq 0$.
- The F firm treats the option as an AD security because
  1. B firms selling $\theta_B(z)$ are risk neutral and have deep pockets.
  2. Options are in zero net supply, $\theta_B(z) + \theta_F(z) = 0$.
  3. $\Longrightarrow$ the option is actuarially fair insurance, $\phi(z) = \pi(z)$.

Krishnamurthy's KM Model: Proposition 2

- Assume the F firm is credit constrained in every state of the world.
  1. $\Longrightarrow \theta_F(z_L) \geq \theta_F(z_H)$.
  2. This holds with strict inequality if $\theta_F(z_L) > 0$.

- Proof by contradiction: suppose not $\Longrightarrow \theta_F(z_L) < \theta_F(z_H)$
  1. However, $\mathcal{W}_{F,1}(z_H) - \mathcal{W}_{F,1}(z_L) \geq [z_H - z_L]\, f(K_{F,0}) > 0$.
  2. The concavity of $\mathcal{J}(\cdot) \Longrightarrow \mathcal{J}'\big(\mathcal{W}_{F,1}(z_L)\big) > \mathcal{J}'\big(\mathcal{W}_{F,1}(z_H)\big)$.
  3. $\Longrightarrow$ the marginal value of another unit of wealth is higher in the low productivity state of the world.
  4. In this case, the F firm values an extra unit of wealth more when $z = z_L$ than otherwise.
  5. Thus, $\theta_F(z_L) > \theta_F(z_H)$ for the option to have positive value.
  6. But if investing in land at date 0 has rate of return dominance, $\theta_F(z_L) = \theta_F(z_H) = 0$.

Krishnamurthy's KM Model: Proposition 2's Implications

- Proposition 2 predicts that $\theta_F(z)$ is inversely related to the state of the world.
- Nonetheless, the collateral constraint on land still binds in this economy $\Longrightarrow$ volatility effect driven by a tighter collateral constraint today results in less output today.
- Options negate the KM model's persistence effect.
- Why? Risk sharing sets $\mathcal{J}'\Big(\mathcal{W}_{F,t}(z_L)\Big) = \mathcal{J}'\Big(\mathcal{W}_{F,t}(z_H)\Big)$.
- B and F firms no longer hold expectations that current changes in the value of collateral drives the value of the collateral constraint and the value of output in the future.

Krishnamurthy's KM Model: Incomplete Hedging

- Implicit in Proposition 2 is that the supply of options, $\theta_B(z)$, is perfectly elastic.
- If $\theta_B(z)$ is in inelastic supply, what is the impact on the KM persistence effect?
- Krishnamurthy assumes that B firms must also post collateral (*i.e.*, land) when selling options to F firms.
- A collateral constraint on options guarantees that the B firms have committed the resources for the option to pay off.
- This matters because the F firms demand more $\theta_B(z)$ in the z_L state of the world $\Longrightarrow$ do B firms have the grain to pay off their debt?

Krishnamurthy's KM Model: Incomplete Hedging

- Assumption 1: (Aggregate collateral). Sale of financial securities must be collateralized by land.
- A B firm faces the collateral constraint

$$\theta_B(z) \;\leq\; -q_1(z)\mathscr{L},$$

  where $\theta_F(z) = -\theta_B(z)$ and $\mathscr{L}$ is the land owned by the B firm.
- Krishnamurthy also assumes that
  1. the B firms own all the land,
  2. which they rent to the F firms, $K_{F,1}(z)$
  3. $\Longrightarrow$ all land is rented.
  4. $\Longrightarrow$ For the options market to clear, $\theta_F(z) = -\theta_B(z) \leq q_1(z)\overline{K}$.
- The date 0 rental price of land is $q_0 - \mathbf{E}\Big\{\phi(z)q_1(z)\,\Big|\,z\Big\}$, which reflects the impact of incomplete hedging during date 0.

Krishnamurthy's KM Model: Incomplete Hedging, cont.

- Incomplete hedging drives a wedge into the price of the option that is a premium on top of the price of actuarial fair insurance.
- Assume the premium, $\eta(z)$, increases $\phi(z)$ by a multiple of $\pi(z)$ under incomplete hedging, or $\phi(z) = \eta(z)\pi(z)$.
- Adjust the options market clearing condition to reflect the insurance premium, $\theta_F\Big(z,\, \eta(z)\Big) \leq q_1(z)\overline{K}$.
- Thought experiment: impact of aggregate collateral constraint?
  1. When $z = z_L$, the F and B firms are credit constrained.
  2. When $z = z_H$, the F and B firms are not credit constrained.
- In the unconstrained state of the world, F and B firms demand the same amount of land at the end of date 0, $K_{F,1}(z_H) = K_{B,1}(z_H)$.
- Market clearing forces $K_{F,1}(z_H) = \frac{1}{2}\overline{K}$ and the DRS technology leads to $q_1(z_H) = \mathcal{A} - \alpha - \overline{K}$.

Krishnamurthy's KM Model: Incomplete Hedging, cont.

- Suppose $z = z_L$ results in F firms having no production at date 0.
- If F firms want to produce in date 1 in the low productivity state of the world, they must buy options, $\theta_F(z_L)$, on $K_{F,1}(z_L)$.
- Remember an unhedged but constrained F firm is restricted by $\mathcal{W}_{F,1}(z) < \left[\alpha + q_1(z)\right]K_{F,1}(z)$, where $\mathcal{W}_{F,1}(z) < \overline{\mathcal{W}}_{F,1}(z)$.
- That is, a constrained F firm's demand for land is less than efficient.
- In the low productivity state of the world, a hedged F firm buys options to mitigate the collateral constraint it faces $\Longrightarrow \theta_F(z_L) = \left[\alpha + q_1(z_L)\right] K_{F,1}(z_L)$.
- A hedged F firm achieves an outcome that is equivalent to having date 1 wealth equal to $\overline{\mathcal{W}}_{F,1}(z)$.

Krishnamurthy's KM Model: Incomplete Hedging, cont.

- Since $zf\Big(K_{F,0}(z)\Big) = 0$ at $z = z_L$, F firms use $\theta_F(z_L)$ to buy grain.
- The state contingent lifetime program facing F firms is

$$\text{Max}_{\{K_{F,0}, K_{F,1}(z_L), K_{F,1}(z_H)\}} \Bigg[\Big(1 - \pi(z_H)\Big) z_H f\Big(K_{F,0}\Big) + f\Big(K_{F,1}(z_H)\Big) - \Big[\alpha + q_1(z_H)\Big] K_{F,1}(z_H) + \pi(z_L) f\Big(K_{F,1}(z_L)\Big) \Bigg]$$

$$\text{s.t.} \qquad K_{F,1}(z_L) = \frac{w_{F,0} - \Big[\alpha + q_0 - \mathbf{E}\Big\{\phi(z) q_1(z) \Big| z\Big\}\Big] K_{F,0}}{\phi(z_L)\Big[\alpha + q_1(z_L)\Big]}.$$

- At date 0 when $z = z_L$, F firms do not produce $\Longrightarrow K_{F,0} = 0$ $\Longrightarrow \Big[\alpha + q_1(z_L)\Big] K_{F,1}(z_L) = \dfrac{w_{F,0}}{\eta(z_L)\pi(z_L)} = \theta\Big(z_L, \eta(z_L)\Big)$.
- A F firm purchases options up to a fraction of its date 0 endowment of grain, which is deflated by the premium $\eta(z_L)$.

Krishnamurthy's KM Model: Incomplete Hedging, cont.

- $\theta_F\Big(z_L,\, \eta(z_L)\Big) = \Big[\alpha + q_1(z_L)\Big]K_{F,1}(z_L)$ represents the demand for options by F firms in the low productivity state of the world.
- B firms supply of options is no more than $q_1(z_L)\overline{K} = \theta_F\Big(z_L,\, \eta(z_L)\Big)$.
- Equate the supply of and demand for options to find equilibrium demand for land by F firms

$$K_{F,1}(z_L) = \frac{q_1(z_L)}{\alpha + q_1(z_L)}\overline{K} \quad \text{or} \quad q_1(z_L) = \frac{\alpha K_{F,1}(z_L)}{\overline{K} - K_{F,1}(z_L)}.$$

- B firms supply land using $q_1(z_L) = \mathcal{A} - \alpha - 2\Big[\overline{K} - K_{F,1}(z_L)\Big]$.
- Two equilibria occur in the market for land when $z = z_L$; see Figure 3.
    1. Solving either for $q_1(z_L)$ or for $K_{F,1}(z_L)$ yields a quadratic.
    2. Existence of the two equilbria rely on $\mathcal{A} - \alpha \leq 2\overline{K}$ and
    3. $q_1(z_L)\overline{K} = \theta_F\Big(z_L,\, \eta(z_L)\Big)$.

Krishnamurthy's KM Model: Incomplete Hedging, cont.

- Suppose that the aggregate collateral constraint is not binding $\Longrightarrow$ F firms generate positive production at date 0.
- A unique equilibrium can be the outcome in which $q_1(z_H) > q_1(z_L)$ $\Longrightarrow$ F firms have standard issue downward sloping demand for land.
- The KM volatility effect is still in play when $z = z_H$, but this shock fails to generate persistence.
  1. F firms insure against bad outcomes at the margin, but
  2. future levels of real activity are unaffected by the z_H state of the world at date 0.
  3. There are no expectations that land prices will fall from date 0 to date 1 $\Longrightarrow$ collateral constraints on B firms are not binding.
- Result is incomplete hedging eliminates persistence in the good state of the world.

Krishnamurthy's KM Model: Incomplete Hedging, cont.

- Persistence occurs in the bad state of the world, $z = z_L$.
- Given $q_1(z_L)\overline{K} = \theta_F\left(z_L,\, \eta(z_L)\right)$, there is "excess demand" for options by F firms that B firms satisfy only by raising $\phi(z_L)$.
  1. F firms insure against bad outcomes at the margin, but
  2. the level of real activity is lower in the z_L state of the world at date 0 (relative to the non-collateral constrained equilibrium).
  3. There is a shortage of grain available for date 1 production.
  4. Expectations are for land prices to fall from date 0 to date 1, which induces tighter collateral constraints on B firms.
  5. F firms will have less grain to plant on land that has fallen in value while the price of options has risen.

Krishnamurthy's KM Model: Summing Up

- Krishnamurthy puts the onus of collateral constraints on lenders rather than borrowers as KM do.
- When a bad shock is realized, lenders withdraw collateral from the market or its value falls sharply.
- The shock is transmitted from lenders to borrowers in the real economy because less financial intermediation takes place in the bad, collateral constrained equilibrium $\Longrightarrow$ there is less collateral or collateral has lower value to support credit creation.
- Whether B firms can default and the impact of these events on the real economy is not addressed by Krishnamurthy.
  1. If B firms default on their options, is this outcome transmitted to the real economy? What is the mechanism?
  2. Should government respond to financial market defaults?
  3. If central banks and Treasuries act during a financial crisis, what form should the responses take?

Holmström and Tirole: Credit Constraints

- Credit constraints impinge on firms in some states of the world, but firms need cash to operate in all states of the world.
- Firms want to smooth access to credit across states of the world $\Longrightarrow$ goal is for credit to be non-state contingent.
- Credit has many sources, but a balance sheet has only two sides.
  1. Existing liabilities provide cash to support credit needs.
     - A. The funds a FI has previously extended to a firm, but the firm has not yet drawn.
     - B. Some firms can issue debt (*i.e.*, commercial paper or corporate bonds).
  2. A firm's assets can be used to loosen credit constraints, but
     - A. these assets must be easy to sell (*i.e.*, liquid) or
     - B. be safe enough to serve as collateral when traded for cash.
     - C. $\Longrightarrow$ Repo agreements in which cash is "rented" in exchange for riskless securities at small discounts (*i.e.* haircuts).

Holmström and Tirole: Liquidity

- Firms use assets and/or liabilities to smooth credit shocks.
- Using assets and liabilities in this way generates a demand for liquidity.
  1. Cash and/or securities (for repo) are necessary to conduct trades.
  2. $\Longrightarrow$ Firms demand liquidity to insure against credit shocks.
  3. Liquidity is insurance against shocks drying up external funds a firm needs to run a project $\Longrightarrow$ aim to equate the price of credit across states of nature,
  4. but credit constraints restrict the insurance to be incomplete $\Longrightarrow$ the price of credit is not equated across states of nature.
  5. Otherwise, credit constraints would not bind, insurance would be complete, and there would be no derived demand for liquidity.
- What are the underlying sources of liquidity demand by firms?
- There are several, but Holmström and Tirole focus on
  1. the inability of firms to pledge project returns to investors.
  2. Not all the income of a project can be credibly promised to investors (outsiders) by the managers (insiders).

Holmström and Tirole: Pledgeability, Credit Constraints, and Liquidity

- There are several ways to model or motivate the demand for liquidity.
  1. Adverse selection $\Longrightarrow$ lemons problem: ex ante acquire funding to run projects that ex post are riskiest and most in need of cash to continue.
  2. Competition for credit $\Longrightarrow$ create incentives for firms to engage in policies that harm the credit quality of competition.
  3. In defense, liquidity is acquire by a firm to insure against shocks to expectations about its credit quality.
- Holmström and Tirole focus on
  1. the inability of firms to pledge project returns to investors.
  2. $\Longrightarrow$ Not all the income of a project can be credibly promised to investors (outsiders) by the managers (insiders).
- Investors are averse to lend to firms when project returns are pledged to back credit repayment.
  1. $\Longrightarrow$ Firms are credit constrained throughout the life of project.
  2. However, credit rationing is different when a firm starts a project compared with a firm trying to obtain credit for an ongoing project.
  3. Nonetheless, firms expect to face credit rationing within the Holmström and Tirole model.

Holmström and Tirole: The Basic Model

- HT assume an investment project has a certain return.
  1. Entrepreneur cannot pledge all the return to investors.
  2. Pledgeable part of return < investment needed to fund the project $\Longrightarrow$ entrepreneur funds shortfall with own resources.
  3. Own funds $\Longleftrightarrow$ Net worth places an upper limit on contributions to project by the entrepreneur.
- An investment project pays off $\mathcal{Z}_1$ (> 0) for a risk neutral entrepreneur
  1. $\Longrightarrow$ a positive net (discounted) present value.
  2. The scale of the project is fixed by assumption.
- The project requires $\mathcal{I}$ as an initial investment, but the entrepreneur can only pledge $\mathcal{Z}_O$ to outside investors $\Longrightarrow$ assume $\mathcal{Z}_1 > \mathcal{I} > \mathcal{Z}_O > 0$.
  1. $\Longrightarrow$ The entrepreneur cannot finance the project without investors.
  2. Pledgeability restriction $\Longrightarrow$ investors cannot/will not fund alone.
  3. $\Longrightarrow \mathcal{I} - \mathcal{Z}_O$ = entrepreneur's contribution to the project.
- Entrepreneur collects "rents" = $\mathcal{Z}_1 - \mathcal{Z}_O > 0 \Longrightarrow$ return to the project
  1. only raises the entrepreneur's welfare (*i.e.*, private benefit) or
  2. falls if entrepreneur is not paid sufficiently (*i.e.*, an externality).
  3. Given $\mathcal{Z}_1 > \mathcal{I}$, if $\mathcal{Z}_1 = \mathcal{Z}_O$, investors are not needed $\Longrightarrow \mathcal{Z}_O > \mathcal{I}$.

Pledgeability as a Financial Friction

- Assume the entrepreneur can commit no more than $\mathcal{A}$ to the project.
  1. *Iff*, pledgeable income $> \mathcal{I} - \mathcal{Z}_O$ does the project happen.
  2. $\Longrightarrow \mathcal{A} \geq \mathcal{I} - \mathcal{Z}_O > 0$ and define $\overline{\mathcal{A}} \equiv \mathcal{I} - \mathcal{Z}_O$.
  3. The entrepreneur needs at least $\overline{\mathcal{A}}$ in net worth to signal to investors that everyone's incentives are aligned.
  4. Otherwise $(\mathcal{A} < \overline{\mathcal{A}})$, the firm is credit constrained.

- Another credit constraint occurs when $\mathcal{A} < \mathcal{Z}_1 - \mathcal{Z}_O$.
  1. $\Longrightarrow$ The entrepreneur is "net worth" poor.
  2. Not all projects are funded for entrepreneurs facing the net worth financial constraint.

- An entrepreneurs having sufficient net worth invests in a project that covers the expenses entailed by its ex post returns.
  1. $\mathcal{Z}_1 - \mathcal{Z}_O - \mathcal{A} \Longrightarrow$ net rents or returns to the entrepreneur given her investment $\mathcal{A}$.
  2. When $\mathcal{A} > \mathcal{Z}_1 - \mathcal{Z}_O$, all projects with $\mathcal{Z}_1 - \mathcal{I} > 0$ operate.
  3. $\Longrightarrow \mathcal{Z}_1 - \mathcal{I} \geq \mathcal{Z}_1 - \mathcal{Z}_O - \mathcal{A}$.

Risk Neutral Preferences and Project Payoffs

- Since the entrepreneur is risk neutral,
  1. a net worth poor entrepreneur, $\mathcal{A} < \overline{\mathcal{A}}$, receives utility of $\mathcal{U}_E = \mathcal{A}$.
  2. Otherwise, $\mathcal{A} \geq \overline{\mathcal{A}} \Longrightarrow \mathcal{U}_E = \mathcal{A} + \mathcal{Z}_1 - \mathcal{I}$.

- There is a discontinuity in entrepreneurial utility at $\mathcal{A} = \overline{\mathcal{A}}$.
  1. Entrepreneurial utility is not a smooth function of the resources committed to the project.
  2. The fixed scale of the project is the source of the discontinuity.

- The discontinuity has implications for the entrepreneur of the value of internal and external investment funds.
  1. When the entrepreneur is credit constrained $\Longrightarrow$ the entrepreneur lacks sufficient net worth to pay investors $\mathcal{Z}_O$ because $\mathcal{A} < \overline{\mathcal{A}}$,
  2. project funding are worth more to the entrepreneur than to investors.
  3. A social planner would transfer funds from investors to entrepreneurs $\Longrightarrow$ more projects funded raises output in the aggregate.
  4. However, this does not raise social welfare $\Longrightarrow$ heterogeneous agents cannot transfer utility without an explicit mechanism for doing so.

Why Assume $\mathcal{Z}_1 - \mathcal{Z}_O > 0$?

- Are HT reasonable to assume $\mathcal{Z}_1 > \mathcal{Z}_O \Longrightarrow$ is a primitive of the economy?
- Expand the model to two periods, $t = 0, 1$.
  1. At $t = 0$, there is a project that costs $\mathcal{I}$ to operate.
  2. The project pays off either 0 or $\mathcal{R}$ at date $t = 1$.
  3. Assume the discount rate = 1.
- The entrepreneur has net worth $\mathcal{A}$ and discretion over where to invest $\mathcal{I}$.
  1. Again, $\mathcal{I} - \mathcal{A} > 0$ and limited liability shields the entrepreneur's net worth from investor claims.
  2. Investors want the entrepreneur to invest in the good project paying $\mathcal{R}$ with probability p_H.
  3. The entrepreneur is tempted by the bad project paying $\mathcal{R}$ with probability $p_L < p_H$, but
  4. the bad project gives the entrepreneur a private benefit $\mathcal{B} < \mathcal{I}$.
- Ex ante returns are > 0 for the good project and < 0 otherwise.
  1. $\Longrightarrow p_H\mathcal{R} - \mathcal{I} > p_L\mathcal{R} - \mathcal{I} + \mathcal{B} \Longrightarrow (p_H - p_L)\mathcal{R} > \mathcal{B}$.
  2. Ex ante investors prefer not to invest in the project if the entrepreneur selects the bad project.

The Project's Payoffs

- If a project is successful, at date $t = 1$ the investor's payoff is
  1. $\mathcal{Y}_S = \mathcal{R} - \mathcal{X}_S$, where the entrepreneur receives $\mathcal{X}_S$ in this state.
  2. Otherwise, the project fails at date $t = 1 \Longrightarrow \mathcal{Y}_F = -\mathcal{X}_F$.
- How can the investor insure the entrepreneur chooses the good project?
- There are two constraints.
  1. The zero profit condition $p_H(\mathcal{R} - \mathcal{X}_S) + (1 - p_H)(-\mathcal{X}_F) \geq \mathcal{I} - \mathcal{A}$.
  2. An ICC, $p_H\mathcal{X}_S + (1 - p_H)\mathcal{X}_F \geq p_L\mathcal{X}_S + (1 - p_L)\mathcal{X}_F + \mathcal{B}$.
  3. $\Longrightarrow$ Creates incentives for the entrepreneur to operate the good project.
- Along with limited liability, the ICC implies $\mathcal{X}_S - \mathcal{X}_F \geq \dfrac{\mathcal{B}}{p_H - p_L} \Longrightarrow$ if the entrepreneur selects the good project, it generates positive income.
- Investor returns are maximized ex post at $\mathcal{X}_S = \dfrac{\mathcal{B}}{p_H - p_L} \Longrightarrow \mathcal{X}_F = 0$.
- Ex ante the entrepreneur can credibly pledge $\mathcal{Z}_O = p_H\left(\mathcal{R} - \dfrac{\mathcal{B}}{p_H - p_L}\right)$ at most to investors $\Longrightarrow \mathcal{Z}_1 - \mathcal{Z}_O = p_H\left(\dfrac{\mathcal{B}}{p_H - p_L}\right) > 0$, assuming $\mathcal{Z}_1 \equiv p_H\mathcal{R}$.

CRS Investment Projects

- A firm's decision to invest is often about the size or scale of the project.
- When it is, there is a trade off between this scale and a firm's decision to accumulate resources to have liquidity to support investment projects in bad states of the world.
- Extend the previous model by having the scale (or cost) of the project be a choice variable, which is denoted $\mathcal{I} \sim$ CRS,
  1. $\Longrightarrow$ as the scale of the project rises, returns are unchanged.
  2. ρ_1 = expected return and ρ_0 = pledgeable income, where $\rho_1 > 1 > \rho_0$.
  3. $\Longrightarrow$ The entrepreneur collects rent of at least $= (\rho_1 - \rho_0)\mathcal{I}$.
  4. $\Longrightarrow$ The map to the fixed investment model is $(\rho_1, \rho_0) = (\mathcal{Z}_1, \mathcal{Z}_0)$.
- Assume $\mathcal{R}\mathcal{I}$ and $\mathcal{B}\mathcal{I}$ are the return to a successful project and the entrepreneur's private benefit $\Longrightarrow \rho_1 = p_H\mathcal{R}$ and $\rho_0 = \rho_1 - \dfrac{p_H\mathcal{B}}{p_H - p_L}$.

Leverage and Variable Investment Projects

- Given $\rho_1 > 1 > \rho_0 > 0$, the firm needs net worth of $\mathcal{A} > 0$ to operate a project of scale $\mathcal{I}$.
  1. $\Longrightarrow$ the pledgeability constraint is $\mathcal{A} \geq (1 - \rho_0)\mathcal{I}$, where
  2. $(1 - \rho_0)\mathcal{I}$ = entrepreneur's own minimum investment in the project.
  3. $\Longrightarrow$ When the constrain binds, flip it to show $\mathcal{I} = \kappa\mathcal{A}$, where $\kappa > 1$.
  4. $\Longrightarrow \kappa \equiv \dfrac{1}{1 - \rho_0}$ is the entrepreneur's leverage per unit of own resources invested in the project.
- The entrepreneur invests in a project with maximum scale $\mathcal{I} = \mathcal{A}\big/(1 - \rho_0)$.
  1. Project payoff per unit of entrepreneurial investment is $\mu = \kappa(\rho_1 - \rho_0)$.
  2. Since $\mu > 1$, the entrepreneur's own return to the project dominates that offered by the market $\Longrightarrow$ underinvestment in the project.
- The net return to or utility of the entrepreneur is $\mathcal{U} = (\mu - 1)\mathcal{A}$, where
  1. $\mu - 1 = \dfrac{\rho_1 - 1}{1 - \rho_0} \Longrightarrow \rho_1 - 1$ is the social gain from moving a unit of resources from investors to the entrepreneur.
  2. This changes the transferred resources into illiquid from liquid assets $\Longrightarrow$ returns from a larger project cannot be pledged to investors.

Credit Constrained Borrowing and Lending

- $\mathcal{A} = (1 - \rho_0)\mathcal{I}$ shows the scale of a project increases with ρ_0.
- $\mathcal{U} = (\mu - 1)\mathcal{A}$ shows larger ρ_1 and ρ_0 raises entrepreneurial utility.
- Per unit of investment, the response of the expected return, ρ_1 to pledgeable income, ρ_0, is $\frac{d\rho_1}{d\rho_0} = 1 - \mu < 0$ [use $\mu = \kappa(\rho_1 - \rho_0)$].
    1. $\Longrightarrow$ an entrepreneur will cease to take on projects with lower ρ_1
    2. at the point where the drop in ρ_1 caused by an increase in ρ_0
    3. is equated to μ (the internal rate of return) net of market return (which is assumed to $= 1$).
    4. The entrepreneur trades project scale and ρ_1 for liquidity.
    5. Investors offer liquidity for more pledgeable income, but
    6. the entrepreneur's net worth constrains a project's pledgeable income.
    7. Finiteness of entrepreneurial net worth is the first part of the HT financial friction $\Longrightarrow$ the return to this net worth $>$ the market return because it is in limited supply.
    8. Entrepreneur attracts investors by pledging project income to them, but this pledge $<$ expected return $\Longrightarrow$ wedge between pledged and non-pledged income.

Liquidity Shocks

- Model liquidity demand as the response to a shock that has the potential to shutdown a project.
- Insert a period between dates $t = 0$ and $t = 1 \Longrightarrow$ relabel $t = 0, 1, 2$.
  1. During (the new) $t = 1$, there is shock that forces the firm to inject more resources into the project.
  2. Assume the firm knows the distribution from which the shock is drawn at $t = 0$, but does not know the $t = 1$ realization ρ.
  3. The restrictions $\rho_1 > 1 > \rho_0$ require $\rho_1 > \rho > \rho_0$, but $\rho \gtreqless 1$.
  4. If $\rho - \rho_0 > 0$, the project has negative income (in PDV) for investors.
  5. The firm see positive expected returns as long as $\rho_1 - \rho > 0$.
  6. $\Longrightarrow$ Investor will not commit more resources for any pledge of income by the firm, but the firm has an incentive to continue the project.
- Key is firm's rent $= \rho_1 - \rho_0 > 0$, which cannot be transferred to investors.
  1. Investors only hold "liquid" claims, ρ_0, on the project.
  2. These securities are AD securities $\Longrightarrow$ investors accept pledgeable income contingent on ρ.
  3. Risk sharing is incomplete because firm's rent is not insurable, which limits liquidity $\Longrightarrow$ this is the second part of the HT financial friction.

The Demand for Liquidity

- Ex ante the demand for liquidity depends on expected size of ρ.
- Assume liquidity is not in sufficient supply for a firm to purchase insurance against large $\rho \implies$ the liquidity constrained equilibrium is second best.
- In this second best equilibrium, the firm balances the scale of the project against the income pledged to investors conditional on entrepreneurial net worth.
- Also, at $t = 0$, the firm balances the scale of the project against the expected size of ρ at $t = 1$.

The Demand for Liquidity: The Example's Assumptions

- Given $\rho = [\rho_L, \rho_H]$, assume $0 \leq \rho_L < \rho_0 < \rho_H < \rho_1 \implies$ if $\rho_0 < \rho$ and $\rho < \rho_1$ a liquidity constrained firm that wants to continue the project.
- Next, f_L and f_H denote the probabilities of $\rho = \rho_L$ and $\rho = \rho_H$.
- Assume $\rho_0 <$ **Min** $\left\{1 + f_L\rho_L + f_H\rho_H,\ \dfrac{1 + f_L\rho_L}{f_L}\right\} < \rho_1$.
  1. $1 + f_L\rho_L + f_H\rho_H$ = expected cost of the project (across both states).
  2. $(1 + f_L\rho_L)/f_L$ = the expected cost if ρ is "too large" in state H.
  3. Expected costs $> \rho_0$. If not, the entrepreneur self-finances the project.
  4. Expected costs $< \rho_1$. Otherwise, the project is not financed.
- The second best equilibrium is a contract between firm and investors.
  1. The contract is a menu giving the scale of the project, $i(\cdot) \leq \mathcal{I}$.
  2. $\implies$ The inequality indicates the project can continue at a size $< \mathcal{I}$.
  3. $\implies i_L = i(\rho_L) \leq \mathcal{I}$ and $i_H = i(\rho_H) \leq \mathcal{I}$.
  4. The contract also sets payments received by investors and the firm.
  5. The project pays investors "liquid" claims $\rho_0 i_j$ and the firm "illiquid" claims $(\rho_1 - \rho_0)i_j$, $j = L, H$, but at $t = 1$ and $j = L$ (H), the investor receives $\rho_0 - \rho_L > 0$ from (gives $\rho_0 - \rho_H > 0$ to) the firm.

Second Best Investment Choices

- The firm's choice variable is its expected illiquid claim on the project.
- The expected illiquid claim = $(f_L i_L + f_H i_H)(\rho_1 - \rho_0) > 0$.
- The expected liquid claim = $f_L(\rho_0 - \rho_L)i_L + f_H(\rho_0 - \rho_H)i_H > 0$
  $\Longrightarrow \rho_0 - \rho_L > 0$ and $\rho_H - \rho_0 > 0$.
- When $\rho = \rho_L$, $i_L = \mathcal{I}$ because $\rho_1 - \rho_L > 0$ and $\rho_0 - \rho_L > 0$
  $\Longrightarrow$ The project is "fully" funded conditional on ρ_L.
- The remaining choices are over i_H and $\mathcal{I} \Longrightarrow$ the trade off facing the firm.
  1. Define $x = i_H/\mathcal{I} \Longrightarrow$ the ratio of (the minimum) invest needed to continue the project given ρ_H to the initial project scale.
  2. The expected cost of continuing i_H is $\overline{\rho}(x) \equiv f_L\rho_L + f_H\rho_H x$.
  3. The firm needs $\mathcal{I} - \mathcal{A}$ in funds at $t = 0$ to start a project, which equals expected liquid claims or $\mathcal{I} - \mathcal{A} = f_L(\rho_0 - \rho_L)i_L + f_H(\rho_0 - \rho_H)i_H$.
  4. $\Longrightarrow \mathcal{I}(x) = \dfrac{\mathcal{A}}{1 + \overline{\rho}(x) - \rho_0(f_L + f_H x)}$, which set the scale of the project equal to the firm's net worth scaled up by the per unit expected cost of continuing the project net of expected pledged income.

Evaluation of the Second Best, I

- At $t = 0$, the firm's expected net utility (of continuing a project) is $\mathcal{U}(x) = [f_L(\rho_1 - \rho_L)i_L + f_H(\rho_1 - \rho_H)i_H]\mathcal{I} = [\rho_1(f_L + xf_H) - 1 - \overline{\rho}(x)]\mathcal{I}(x)$.
- Substitute for $\mathcal{I}(x) \Longrightarrow \mathcal{U}(x) = [\mu(x) - 1]\mathcal{A} \Longrightarrow$ the firm's expected net utility is a fraction of its contribution to the project, where the marginal value of an extra unit of firm net worth is $\mu(x) = \dfrac{(\rho_1 - \rho_0)(f_L + f_H x)}{1 + \overline{\rho}(x) - \rho_0(f_L + f_H x)}$.
- Note firm utility is linear in $\mathcal{I}(x)$ (or $\mathcal{A}$) $\Longrightarrow$ can think as x as either 0 or 1 $\Longrightarrow$ continue only if ρ is small or always continue $\Longrightarrow i(\rho) = \mathcal{I}$.
  1. $\mathcal{U}(1) - \mathcal{U}(0) = [\mu(1) - \mu(0)]\mathcal{A} \Longrightarrow \mu(0) \leq \mu(1)$, *iff* $x = 1$.
  2. Which is $\dfrac{(\rho_1 - \rho_0)f_L}{1 + (\rho_L - \rho_0)f_L} \leq \dfrac{\rho_1 - \rho_0}{1 + \overline{\rho}(1) - \rho_0}$.
  3. If $\rho_H = \rho_0$, $\mu(0) < \mu(1) \Longrightarrow$ the project continues because pledged income is sufficient to cover the liquidity shock.
  4. As $\rho_H \longrightarrow \rho_1$ from ρ_0, $\mu(0) - \mu(1)$ rises monotonically from $\mu(0) < \mu(1)$ to $\mu(0) > \mu(1) \Longrightarrow$ there is a ρ_H yielding $\mu(0) = \mu(1)$.
  5. Define the "cutoff" c such that $\rho_0 < c < \rho_1 \Longrightarrow$ optimal to continue a project *iff* $\rho_H \leq c$.

Evaluation of the Second Best, II

- The parameter c bounds (from above) the largest value of the "bad" liquidity shock, ρ_H, at which the project is funded at $t = 1$, where $\rho_0 < c < \rho_1$.
- At $\rho_H = c$, the marginal project continues $\Longrightarrow$ c is its per unit cost.
- When $\rho_H < c$, infra-marginal projects continue $\Longrightarrow$ c is per unit cost of an ongoing investment.
- Use $\mu(0) \leq \mu(1)$, or $\dfrac{(\rho_1 - \rho_0)f_L}{1 + (\rho_L - \rho_0)f_L} \leq \dfrac{\rho_1 - \rho_0}{1 + \overline{\rho}(1) - \rho_0}$, $x = 1$, and $\overline{\rho}(x) \equiv f_L\rho_L + f_H\rho_H x$, to show $c = \textbf{Min}\left\{1 + f_L\rho_L + f_H\rho_H,\ \dfrac{1 + f_L\rho_L}{f_L}\right\}$.
  1. When $\rho = \rho_H < c$, continue project *iif* $c = 1 + f_L\rho_L + f_H\rho_H$.
  2. Can show $\mu(0) \leq \mu(1) \Longrightarrow (\rho_H - \rho_L)f_L \leq 1$, which is a necessary and sufficient for the project to continue in either state.
  3. If ρ_L rises, ex ante the firm increases $\mathcal{I}$ and same if ρ_H falls.
  4. Larger $\mathcal{I}$ is needed to support greater expected liquidity costs at $t = 1$ and a smaller ρ_H suggests less liquidity is to back larger $\mathcal{I}$.
  5. Since $f_L > \epsilon > 0$, $\rho_1 - \rho_0$ is positive when $j = L$ or $j = H$.
  6. The firm demands liquidity ex ante, which is managed by making the firm's balance sheet (*i.e.*, assets to liabilities) a choice variable.

Aggregate Liquidity

- Credit rationing remains a primitive of the firm's technology $\Longrightarrow$ firm's problem is largely unchanged.
- There are risk neutral consumers with utility $\mathcal{U}_C = c_0 + c_1 + c_2$.
  1. Consumers own grain, but there is no technology to store the grain $\Longrightarrow$ no way to transfer grain intertemporally.
  2. Consumers supply labor to firms.
- Firms own an intratemporal technology requiring labor and grain to generate more grain, which is consumed when produced.
- Credit rationing remains a primitive of the firm's technology.
  1. At $t = 0$, consumers only commit to invest in the firms' future production plans if firms pledge future output to consumers.
  2. Consumer endowments are not pledged.
  3. $\Longrightarrow$ The pledge is of future income by firms in anticipation of producing at $t = 0, 1, 2$.

A Simple Example

- Problem is unchanged, except at $t = 1$ the firm needs $\rho i(\rho)$ to continue its project, which is non-stochastic, $i(\rho) < \mathcal{I}$, and ρ = liquidity shock.
  1. Assume $\rho > \rho_0$ and $1 + \rho < \rho_1 \Longrightarrow$ guarantees project's PDV > 0.
  2. A firm's net worth $\mathcal{A}$ is put into a project at $t = 0$.
  3. The project is chosen to have an initial scale $\mathcal{I}$, which requires an investment of $(\rho - \rho_0)\mathcal{I} > 0$.
  4. Firms issue liquid assets at $t = 0$, where $(1 + \rho - \rho_0)\mathcal{I} = \mathcal{A}$ is the budget constraint of the project.
  5. At $t = 1$, firms pay for the deterministic liquidity shock, ρ, with the claims issued at $t = 0$, $\rho_0\mathcal{I}$.
  6. These claims are liabilities of firms because $(\rho_0 - \rho)\mathcal{I} < 0$, which are the pledgeable income households invested in projects.

A Simple Example: Implications

- The liquid asset called pledgeable income is the "technology" households use to transfer consumption intertemporally.
- As long as pledgeable income, ρ_0, is large enough households will supply sufficient liquid assets to firms.
- The issues are whether households
  1. are willing to supply pledgeable income (*i.e.*, liquidity) to firms given the primitives of the economy, and
  2. are endowed with enough grain to provide the liquidity firms demand.
- Credit rationing is still a primitive of the economy $\Longrightarrow$ the preferences and technologies of firms.
- There are two frictions generating credit rationing.
  1. First, only part of the expected returns of a project are pledgeable,
  2. which acts as a constraint on the liquid claims firms can offer households in return for pledgeable income $\Longrightarrow$ there is no price at which households will hold these liabilities of firms.
  3. Second, the finiteness of household endowments at date t limits the liquidity households are willing to promise firms at date $t+1$.

Inside Liquidity: Aggregate and Idiosyncratic Shocks

- Alter the previous deterministic example by including
  1. a date $t = 1$ stochastic liquidity shock, which is common to all firms.
  2. An aggregate liquidity shock that is low, ρ_L, or high, ρ_H, $\rho_L < \rho_0 < \rho_H$, with probabilities f_L and f_H, respectively.
  3. When $\rho = \rho_L$, all projects continue given $\rho_1 - \rho_L > 0 \Longrightarrow$ expected returns on projects net of pledged income are positive.
  4. All projects fail in the bad state, $\rho = \rho_H$, because there is insufficient liquidity to satisfy the demand for funds by firms $\Longrightarrow$ all projects fail.
  5. Firms could issue an unlimited amount of liabilities (*i.e.*, pledged income), but household would not agree to issue an equal amount of liquid assets when endowments are in fixed supply.

- Alter the previous deterministic example by including
  1. firm specific idiosyncratic shocks $\Longrightarrow$ appeal to LLN that in aggregate these shocks can be fully insured.
  2. This requires a continuum of ex ante identical firms with unit mass.
  3. Assume firms issue liquid claims to households at $t = 0$ in exchange for pledgeable income at $t = 1 \Longrightarrow$ liquid claims are liabilities of firms.
  4. Sufficient liquidity for firms in aggregate, but a subset of firms fail $\Longrightarrow$ similar to costly state verification problem.

Inside and Outside Liquidity: An Example I

- Return to the previous example of an aggregate liquidity shock.
  1. There is a continuum of ex ante identical firms.
  2. A common liquidity shock afflicts all firms $\Longrightarrow$ $\rho = [\rho_L, \rho_H]$.
  3. Assume $\rho_L < \rho_0 < \rho_H \Longrightarrow$ all projects fail when $\rho = \rho_H$.
  4. Firms issue too few liquid assets to households at $t = 0$ to continue any project in the H state of the world at $t = 1 \Longrightarrow$ insufficient liquidity.

- Add another liquid asset to the economy $\Longrightarrow$ a grain producing technology.
  1. The non-state contingent technology generates $\mathcal{O}$ units of grain per period $\Longrightarrow$ a metaphor for a government liability backed by tax revenue.
  2. Let q denote the price of liquidity per unit $\Longrightarrow$ firms buy a non-state contingent call on some $\mathcal{O}$ at $t = 0$ that is delivered at date $t = 1$.
  3. There are two sources of liquidity in the economy $\Longrightarrow$ q is the relative price of liquid securities issued by firms to $\mathcal{O}$.
  4. When $q = 1$, $\mathcal{O}$ is large $\Longrightarrow$ private and government liquidity are perfect substitutes because there is sufficient liquidity no matter $\rho_0 - \rho_H < 0$.
  5. When $q > 1$, firms are short liquidity $\Longrightarrow$ demand by firms to transfer liquidity intertemporally from $t = 0$ to $t = 1$.
  6. $\Longrightarrow$ Liquidity premium $= q - 1$ at $t = 0$, but $q = 1$ at $t = 1$.

Inside and Outside Liquidity: An Example II

- The existence of inside and outside liquidity gives firms an additional choice variable besides the $t = 0$ initial scale of a project, $\mathcal{I}$, and the $t = 1$ continuation investment $i(\rho_H) \leq \mathcal{I}$.

- The new margin is the liquidity, ℓ_d, a firm buys at $t = 0$.
  1. When $\rho = \rho_L$, $i(\rho_L) = \mathcal{I} \Longrightarrow$ at $t = 1$, a firm continues with its initial plans no matter the ℓ_d purchased at $t = 0$.
  2. Not true for $\rho = \rho_H \Longrightarrow$ a firm uses ℓ_d to continue $i(\rho_H) \leq \mathcal{I}$.
  3. A firm experiences a short fall in project funding of $\rho_H - \rho_0$ per unit of $i(\rho_H) \Longrightarrow (\rho_H - \rho_0)i(\rho_H) \leq \ell_d$ is the firm's liquidity constraint.

- A firm purchasing liquidity adds to its liabilities and to project costs $\Longrightarrow$ these costs in net $= (q - 1)\ell_d \Longrightarrow$ pay q at $t = 0$ for ℓ_d at $t = 1$.

- These costs also alter a firm's budget constraint to

$$\mathcal{I} - \mathcal{A} + (q - 1)\ell_d \ \leq \ f_L(\rho_0 - \rho_L)\mathcal{I} + f_H(\rho_0 - \rho_H)i(\rho_H).$$

- A firm chooses $\mathcal{I}$, $i(\rho_H)$, and ℓ_d at $t = 0$, subject to its budget and liquidity constraints and given q.

Inside and Outside Liquidity: An Example III

- Outside liquidity gives a firm additional choices about $i(\rho_H)$.
  1. The firm could set $i(\rho_H) = 0 \Longrightarrow \mathcal{I}(i_H = 0) = \dfrac{\mathcal{A}}{1 - f_L(\rho_0 - \rho_L)}$.
  2. Or $0 < i(\rho_H) \leq \mathcal{I}$. Consider $i(\rho_H) = \mathcal{I}$,
  3. which gives $\mathcal{I}(q > 1) = \dfrac{\mathcal{A}}{1 - f_L(\rho_0 - \rho_L) - (f_H + q - 1)(\rho_0 - \rho_H)}$, where the liquidity constraint holds with equality.
- The firm's net payoffs under the two plans are
  1. $\mathcal{U}(i_H = 0) = f_L(\rho_1 - \rho_0)\mathcal{I}(i_H = 0) - \mathcal{A} = \left[\dfrac{f_L(\rho_1 - \rho_L) - 1}{1 - f_L(\rho_0 - \rho_L)}\right]\mathcal{A}$.
  2. $\mathcal{U}(q > 1) = \left[\dfrac{\rho_1 - (1 + (q-1)(\rho_H - \rho_0) + \overline{\rho})}{1 + (q-1)(\rho_H - \rho_0) + \overline{\rho} - \rho_0}\right]\mathcal{A}$, where, at $t = 0$, the expected $t = 1$ liquidity shock is $\overline{\rho} \equiv f_L\rho_L + f_H\rho_H$.
  3. If $\mathcal{U}(q > 1) > \mathcal{U}(i_H = 0)$, the project continues in state of the world H.
  4. Assuming $f_L(\rho_H - \rho_L) < 1$, $\mathcal{U}(q = 1) > \mathcal{U}(i_H = 0) \Longrightarrow$ when liquidity is in "infinite" supply and $\rho = \rho_H$, optimal policy is to continue.
  5. As $q \uparrow$, $\mathcal{U}(q > 1) \downarrow \Longrightarrow$ there is a q_{max} where $\mathcal{U}(q_{max}) = \mathcal{U}(i_H = 0)$.
  6. q_{max} is the price of liquidity at which the firm is indifferent between

Inside and Outside Liquidity: An Example IV

- Recover q_{max} by setting $\mathcal{U}(q_{\text{max}}) = \mathcal{U}(i_H = 0)$.

- Suppose $\mathcal{U}(q > 1) = \mathcal{U}(i_H = 0) \Longrightarrow$

$$1 + (q - 1)(\rho_H - \rho_0) + \overline{\rho} \;=\; \frac{1 + \rho_L f_L}{f_L} + \rho_0(f_L + f_H - 1),$$

  1. where the effective unit cost of investment $= 1 + (q - 1)(\rho_H - \rho_0) + \overline{\rho}$ if the firm continues the project no matter the liquidity state,
  2. but if the project continues only in the L liquidity state the effective unit cost $= (1 + \rho_L f_L)/f_L \Longrightarrow$ expected liquidity cost of a unit investment scaled by the probability of successful projects, and
  3. in this case the firm has no need of investors ($i.e.$, $\rho_0 = 0$) because there is ample liquidity $\Longrightarrow$ the expected cost of a unit of pledged income in the L and H liquidity states $= \rho_0(f_L + f_H - 1)$.

- Thus, $\mathcal{U}(q_{\text{max}}) = \mathcal{U}(i_H = 0) \Longrightarrow 1 + (q - 1)(\rho_H - \rho_0) + \overline{\rho} = (1 + \rho_L f_L)/f_L$ or $c(q_{\text{max}}) = c(i_H = 0) \Longrightarrow q_{\text{max}}$ equalizes the costs per unit of investment across the two investment plans.

Inside and Outside Liquidity, V

- Construct aggregate liquidity demand $\mathscr{L}_D = \int \ell_d(j)dj$ using a project's liquidity constraint, $(\rho_H - \rho_0)i(\rho_H) \leq \ell_d$, and budget constraint

$$\mathcal{I} - \mathcal{A} + (q-1)\ell_d \ \leq \ f_L(\rho_0 - \rho_L)\mathcal{I} + f_H(\rho_0 - \rho_H)i(\rho_H).$$

  1. $\Longrightarrow \mathcal{I} = \dfrac{\mathscr{L}_D}{\rho_H - \rho_0}$ and $\left[\dfrac{1}{\rho_H - \rho_0} + q - 1 + \dfrac{f_L(\rho_0 - \rho_L)}{\rho_H - \rho_0} + f_H\right]\mathscr{L}_D = \mathcal{A}$.
  2. $\Longrightarrow \mathscr{L}_D(q) = \dfrac{(\rho_H - \rho_0)\mathcal{A}}{1 + (q-1)(\rho_H - \rho_0) + \overline{\rho} - (f_L + f_H)\rho_0}$.
  3. $\mathscr{L}_D(q) = 0$ when $q > q_{\max} \Longrightarrow$ otherwise planning to continue in both liquidity states costs $>$ expected costs of continuing only in the L state.
  4. $\Longrightarrow \mathscr{L}_D(q_{\max})$ is consistent with $i(\rho_H) \in [0, \mathcal{I}]$.

- Remember liquidity is in fixed supply, $\mathcal{O}$, not infinite supply $\mathcal{O} < \infty$ $\Longrightarrow \mathcal{O} = \mathscr{L}_D(q_{\max})$, which restricts the liquidity premium

$$q_{\max} - 1 \ = \ \frac{\mathcal{A}}{\mathcal{O}} - \frac{1 + \overline{\rho} - (f_L + f_H)\rho_0}{\rho_H - \rho_0}.$$

- The liquidity premium = aggregate shortage of project funds net of expected net unit costs scaled by the liquidity needed when $\rho = \rho_H$.

Inside and Outside Liquidity: Two Comparative Static Experiments

- Suppose $\mathcal{O}$ falls $\Longrightarrow$ q rises. Why?
- A drop in $\mathcal{O}$ makes liquidity scarce, which is the source of an increase in q.
  1. Less and more expensive liquidity $\Longrightarrow$ smaller projects initially, $\mathcal{I}$ falls as q rises to $q_{\max}$ until $q = q_{\max}$.
  2. As $q > q_{\max}$, projects continue only in the L liquidity state $\Longrightarrow$ $\mathcal{I}$ is fixed in size and non-monotone in $\mathcal{O}$.
- Let $\mathcal{A}$ increase $\Longrightarrow$ shift of (not a movement along the) $\mathscr{L}_D(q)$ schedule.
- The shift is proportional (because $\mathcal{A}$ enters the project budget constraint linearly) and only on the part of the $\mathscr{L}_D(q)$ schedule where $q < q_{\max}$.
  1. $\Longrightarrow$ q rises because the aggregate demand for liquidity rises.
  2. At $q = q_{\max}$, with firms indifferent between continuing in the H state or not, $\mathscr{L}_D(q)$ falls because less projects continuing in the H state.
  3. HT interpret this comparative static exercise as suggesting a boom-bust liquidity cycle.

Inside and Outside Money in a NKDSGE Model

- Meh and Moran (MM) graft a "double sided" moral hazard problem into a standard medium scale NKDSGE model.
- Moral hazard is a problem because investors lack a technology to monitor entrepreneurs $\Longrightarrow$ investors delegate monitoring to FIs.
  1. FI monitoring of entrepreneurs is imperfect $\Longrightarrow$ there are costs to monitor and asymmetric information $\Longrightarrow$ one source of moral hazard in the financial market.
  2. Imperfect monitoring by FIs forces the risk of FI loan to entrepreneurs onto the FI's balance sheet $\Longrightarrow$ second source of moral hazard in the financial market.
  3. Investors aims to solve the moral hazard problem they face by requiring FIs to put some of their net worth into investment projects.
  4. Similarly, FIs respond by having entrepreneurs place some of their net worth in their projects.
- Changes in FI net worth alter the "FI loan production possibility frontier."
  1. Less FI net worth shrinks the supply of credit and raises the loan rate,
  2. which is a movement along the credit supply schedule, all else equal.

The Meh and Moran Financial Friction-NKDSGE Model, I

- There are entrepreneurs, FIs, and households whose mass integrate to one.
- There are capital, intermediate, and final goods.
  1. Entrepreneurs produce capital goods using a storage technology subject to idiosyncratic shocks.
  2. Intermediate goods firms produce using a CRS technology afflicted by a fixed cost and Calvo staggered prices $\Longrightarrow$ Intermediate goods are sold to final goods firms in monopolistically competitive markets.
  3. Final goods sold to households in perfectly competitive market by final goods firms that just aggregate intermediate goods.
- Households supply differentiated labor services and rent capital to intermediate goods firms, hold a portfolio of fiat currency and FI deposits, and face consumption habit when consuming the final good.
- A monetary authority operates an interest rate rule, but no fiscal policy.
  1. Money demand is not defined $\Longrightarrow$ fiat currency is in perfectly elastic supply at the given policy rate.
  2. There is no government debt.
  3. What is the supply of outside liquidity?

The Meh and Moran Financial Friction-NKDSGE Model, II

- The financial frictions are derived from costly monitoring and asymmetric information $\Longrightarrow$ moral hazard.
- Investors cannot perfectly monitor FIs and FIs cannot perfectly monitor entrepreneurs.
- MM assume FIs require entrepreneurs to put their new worth, n_t, into projects and FI net worth, a_t, has to be invested in entrepreneurial projects.
- FIs and entrepreneurs are risk neutral,
  1. rent capital and supply labor to intermediate goods firms.
  2. n_t and a_t are the sum of this rental and labor income
  3. plus the market value of their net capital.

The Meh and Moran Financial Friction-NKDSGE Model, III

- The FI-entrepreneur financial contract solves the problem that $n_t < i_t$, where i_t = scale of the project $\Longrightarrow$ entrepreneurs need $i_t - n_t$ from FIs.
- FIs receive d_t in deposits from households (*i.e.*, the investors) $\Longrightarrow a_t + d_t$ = funds available to a FI to loan to entrepreneurs.
- MM only examine one-period debt contracts $\Longrightarrow$ assume FIs and entrepreneurs are randomly matched at each date t.
  1. At $t+1$, new matches are randomly created for FIs and entrepreneurs.
  2. The contract sets i_t, a_t, d_t, and the share of project income received by households, $R_{H,t}$, the FI, $R_{FI,t}$, and an entrepreneur, $R_{E,t}$.
  3. Capital goods project technology is linear and subject to idiosyncratic shocks $= Ri_t$, where $\text{Prob}(R > 1) = \alpha_g$, $\text{Prob}(R = 0) = \alpha_b$, R is *IID* across entrepreneurs, and $\alpha_b < \alpha_g$.
  4. There are three types of projects: (*i*) a successful project with no private benefit for entrepreneurs, (*ii*) an unsuccessful project with private benefit = bi_t, and (*iii*) a second unsuccessful project with private benefit = Bi_t, where $b < B$.
  5. FI monitoring costs = μi_t and although $\mu > 0$ is known to investor, whether FIs monitor entrepreneurs is unknown by investors.

The Meh and Moran Financial Friction-NKDSGE Model: The Financial Contract

- The FI offers an entrepreneur to maximize the expected return on a project.
  1. $\mathbf{Max}_{\{i_t,\, a_t,\, d_t,\, R_{H,t},\, R_{FI,t},\, R_{E,t}\}}\ q_t \alpha_g R_{E,t} i_t$, subject to
  2. $q_t \alpha_b R_{E,t} i_t + q_t b i_t \leq q_t \alpha_g R_{E,t} i_t \implies$ ICC FIs impose on entrepreneurs,
  3. $q_t \alpha_b R_{FI,t} i_t \leq q_t \alpha_g R_{FI,t} i_t - \mu i_t \implies$ ICC investors impose on FIs,
  4. $(1 + r_{a,t}) a_t \leq q_t \alpha_g R_{FI,t} i_t \implies$ FIs are no worse off lending,
  5. $(1 + r_{d,t}) d_t \leq q_t \alpha_g R_{H,t} i_t \implies$ households are no worse off leaving deposits with FIs,
  6. $d_t - (n_t - i_t) \leq a_t - \mu i_t \implies$ FI net worth net of monitoring costs is no less than its liabilities net of assets,
  7. $R = R_{H,t} + R_{FI,t} + R_{E,t}$, income of a successful project is distributed,
  8. where q_t = the relative price of capital (per unit of final goods), $r_{a,t}$ = return to FI net worth, and $r_{d,t}$ = return on deposits.
  9. $\implies$ The equilibrium contract is linear.

The Meh and Moran Financial Friction-NKDSGE Model: Risk and Liquidity

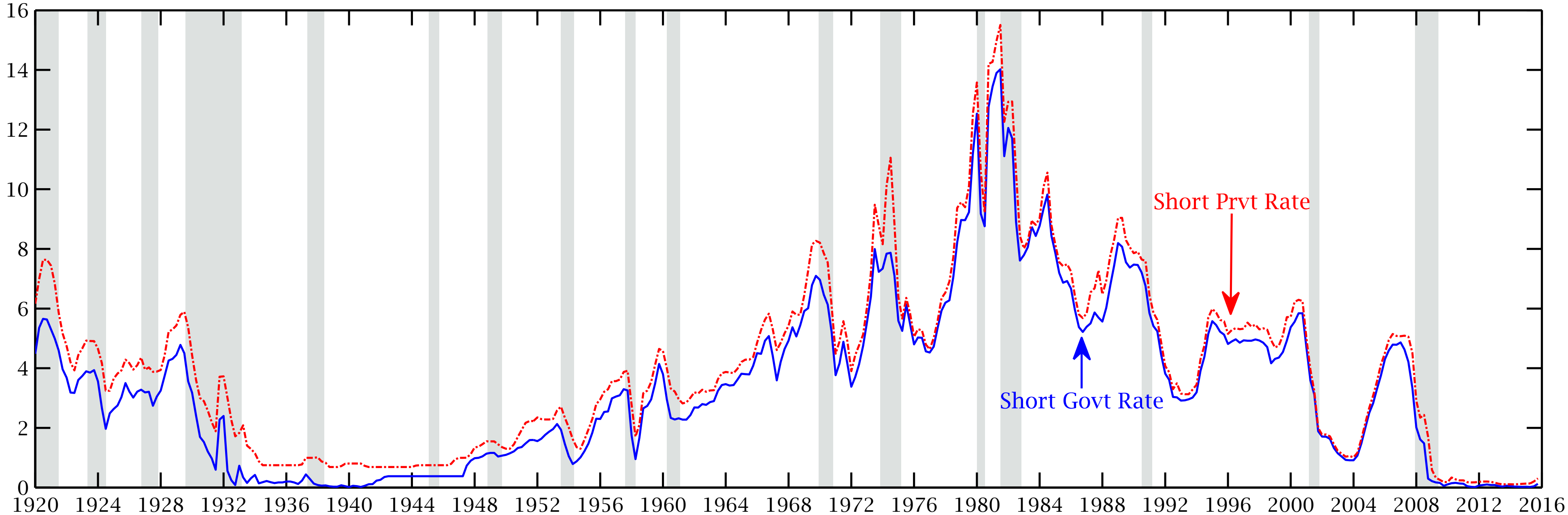
- Consider contracts that satisfy interior solutions.
  1. The entrepreneur's ICC $\Longrightarrow R_{E,t} = b/(\alpha_g - \alpha_b)$.
  2. The FI's ICC $\Longrightarrow R_{FI,t} = \mu/[q_t(\alpha_g - \alpha_b)]$.
  3. These returns and the return adding up condition $\Longrightarrow R_{H,t} = R - [q_t b + \mu]/[q_t(\alpha_g - \alpha_b)]$.
- FIs and households are subject to risk on their balance sheets $\Longrightarrow$ the risk is to the price of capital, which is a macro risk.
- Entrepreneurs are fully insured against this macro risk as long as they invest in successful projects.
- The liquidity risk falls on households and FIs.
  1. Entrepreneurs need FIs to act as outside investors, but
  2. there is no credit rationing ex post $\Longrightarrow$ entrepreneurs are fully insured against idiosyncratic and aggregate shocks.
  3. The return on deposits is a kind of liquidity premium on projects $\Longrightarrow r_{d,t} = \alpha_g R - \left[\dfrac{q_t b + \mu}{\alpha_g - \alpha_b}\right]\dfrac{i_t}{d_t}$.
  4. Liquidity premium = the cost of entrepreneurial and FI ICCs scaled by the net probability of project success times the scale of projects relative to deposits.

The Meh and Moran Financial Friction-NKDSGE Model: Summary

- MM study the dynamics of their financial frictions-NKDSGE model by linearizing around a steady state.
- Also, MM's model has a business cycle propagation mechanism tied to the response of the price of capital, q_t to financial shocks.
- This model does not generate financial crises or boom-bust credit cycles $\Longrightarrow$ there are financial or credit cycles.
- There are no financial crisis produced by MM's model because
  1. it has a steady state $\Longrightarrow$ business and credit cycles are the transitions paths back to this steady state.
  2. This suggests a DSGE model generating financial crisis will need to have multiple equilibria and stochastic steady states.

Figure Rates

Figure: Short and Long Government and Private Interest Rates, 1920Q1 to 2015Q4



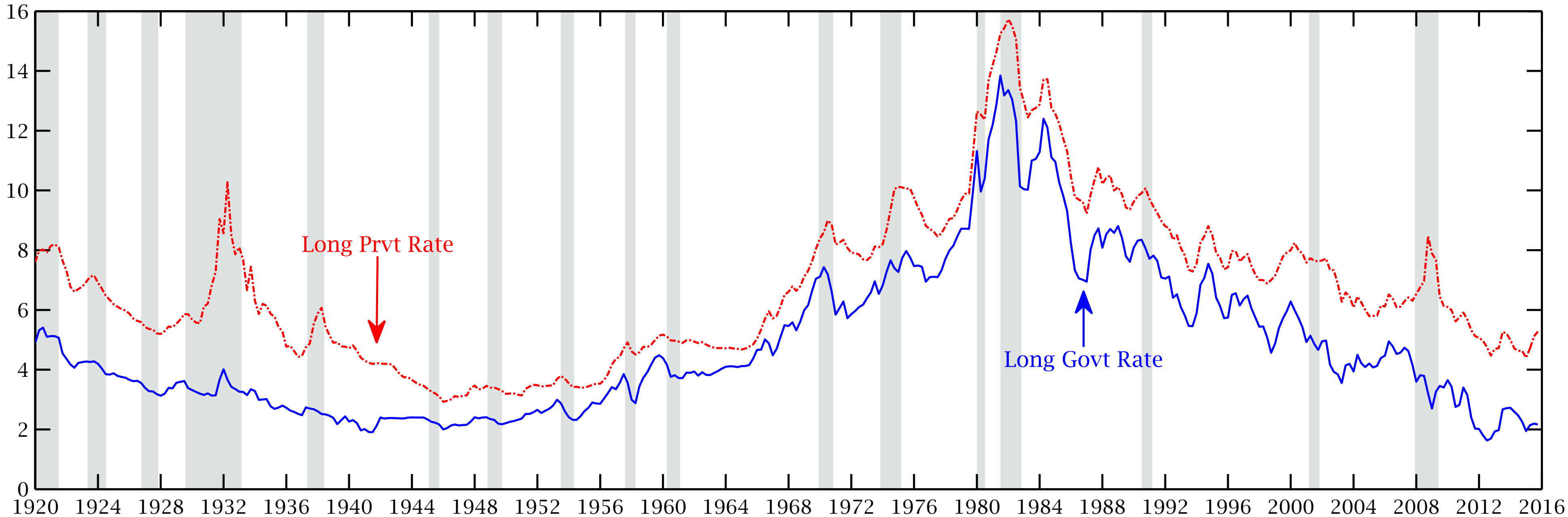


Figure: Short and Long Private-Government Interest Rate Spreads, 1920Q1 to 2015Q4

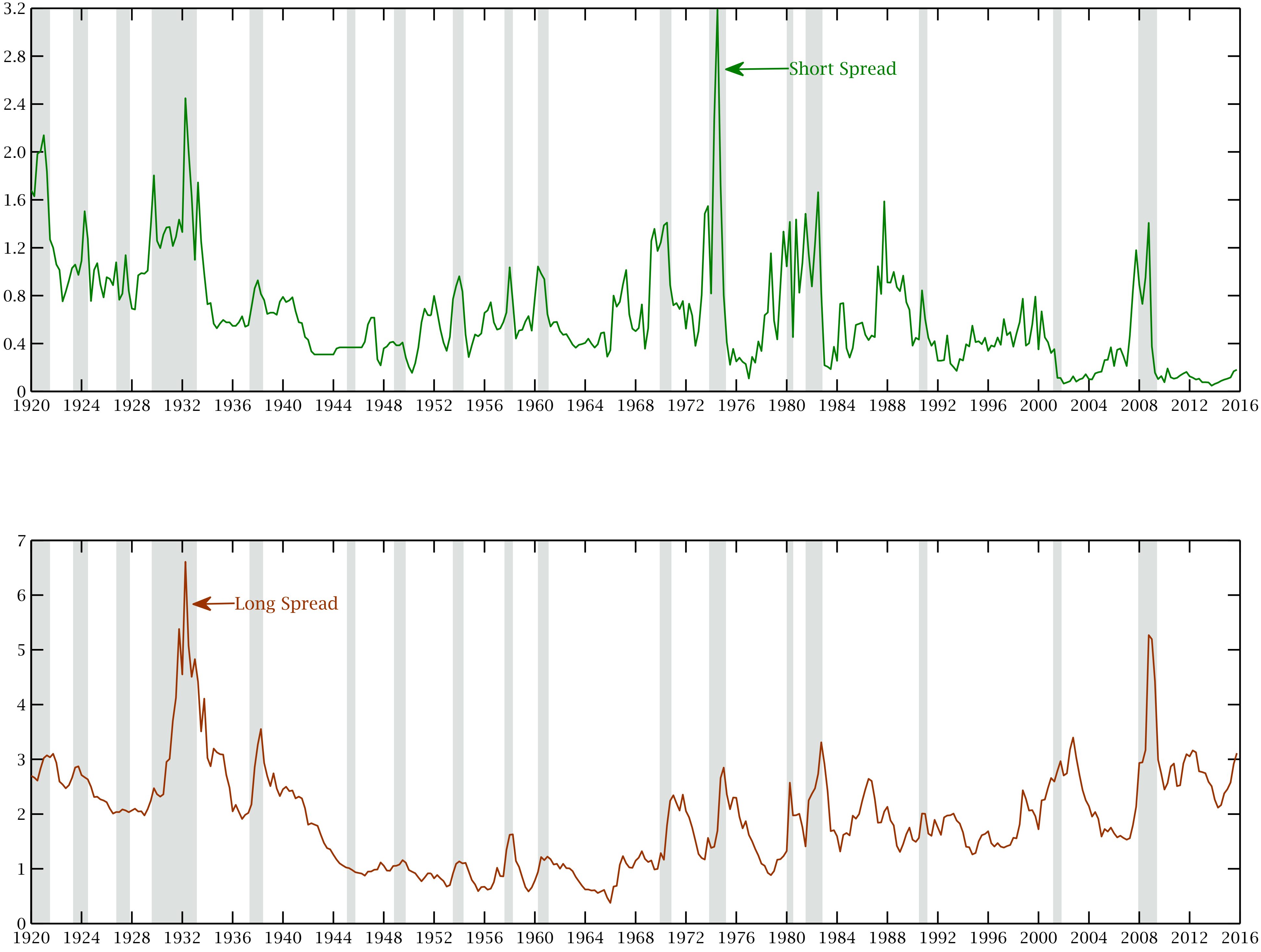
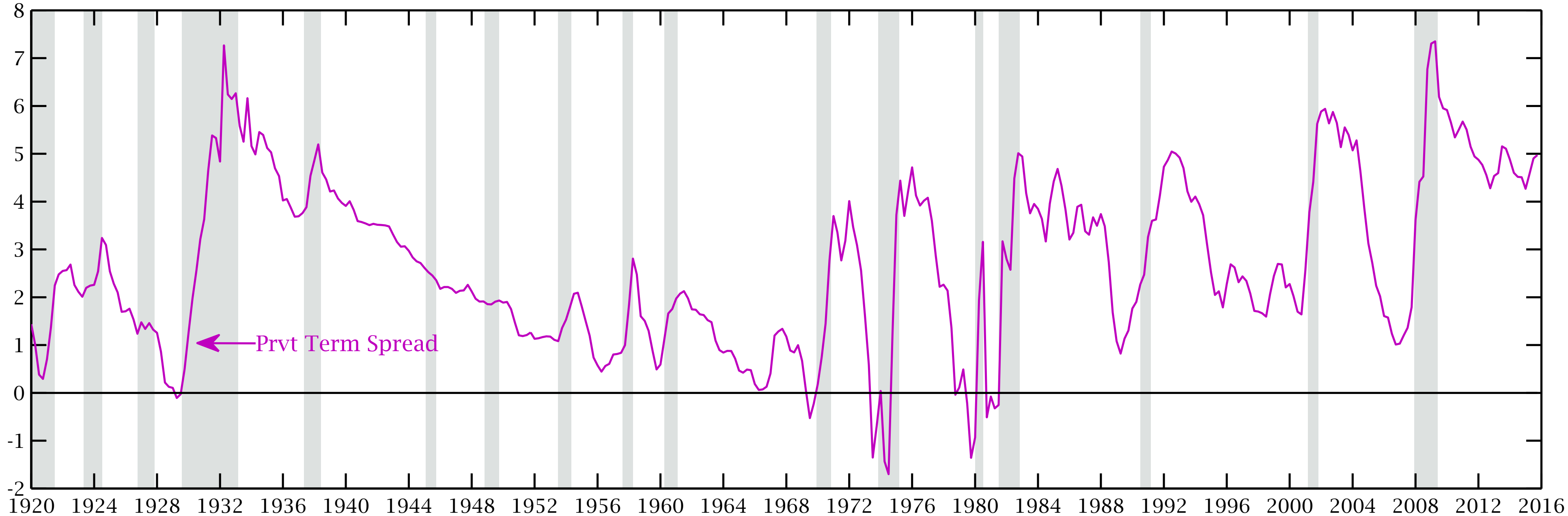
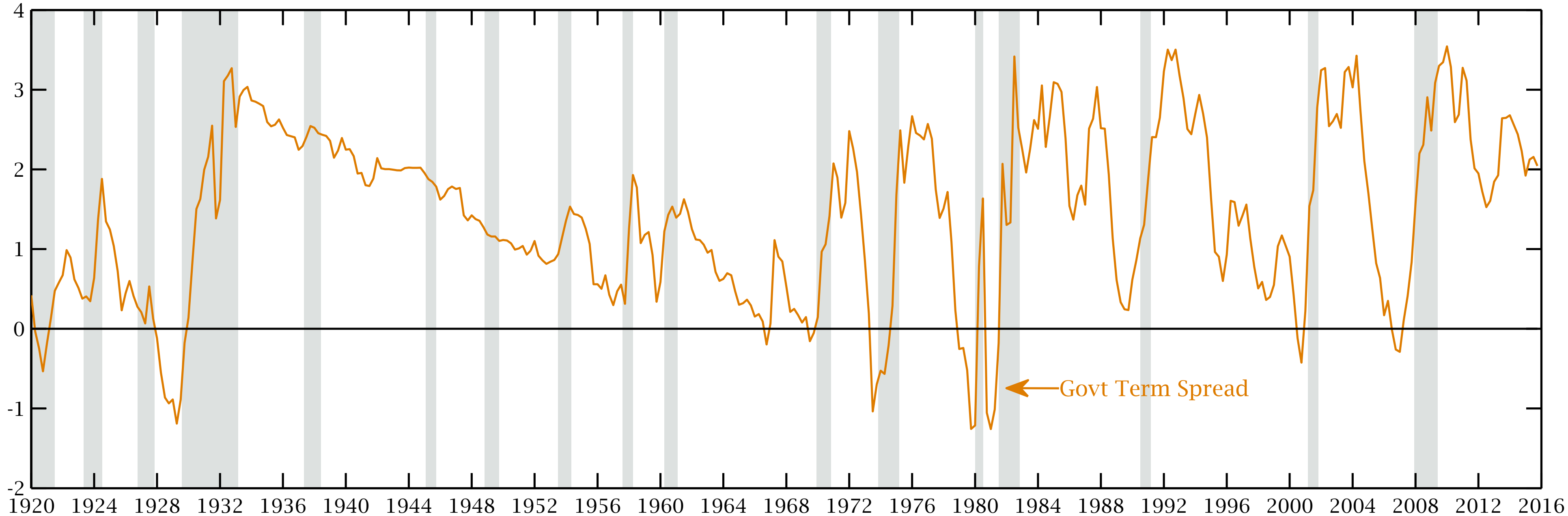


Figure: Private and Government Interest Rate Term Spreads, 1920Q1 to 2015Q4





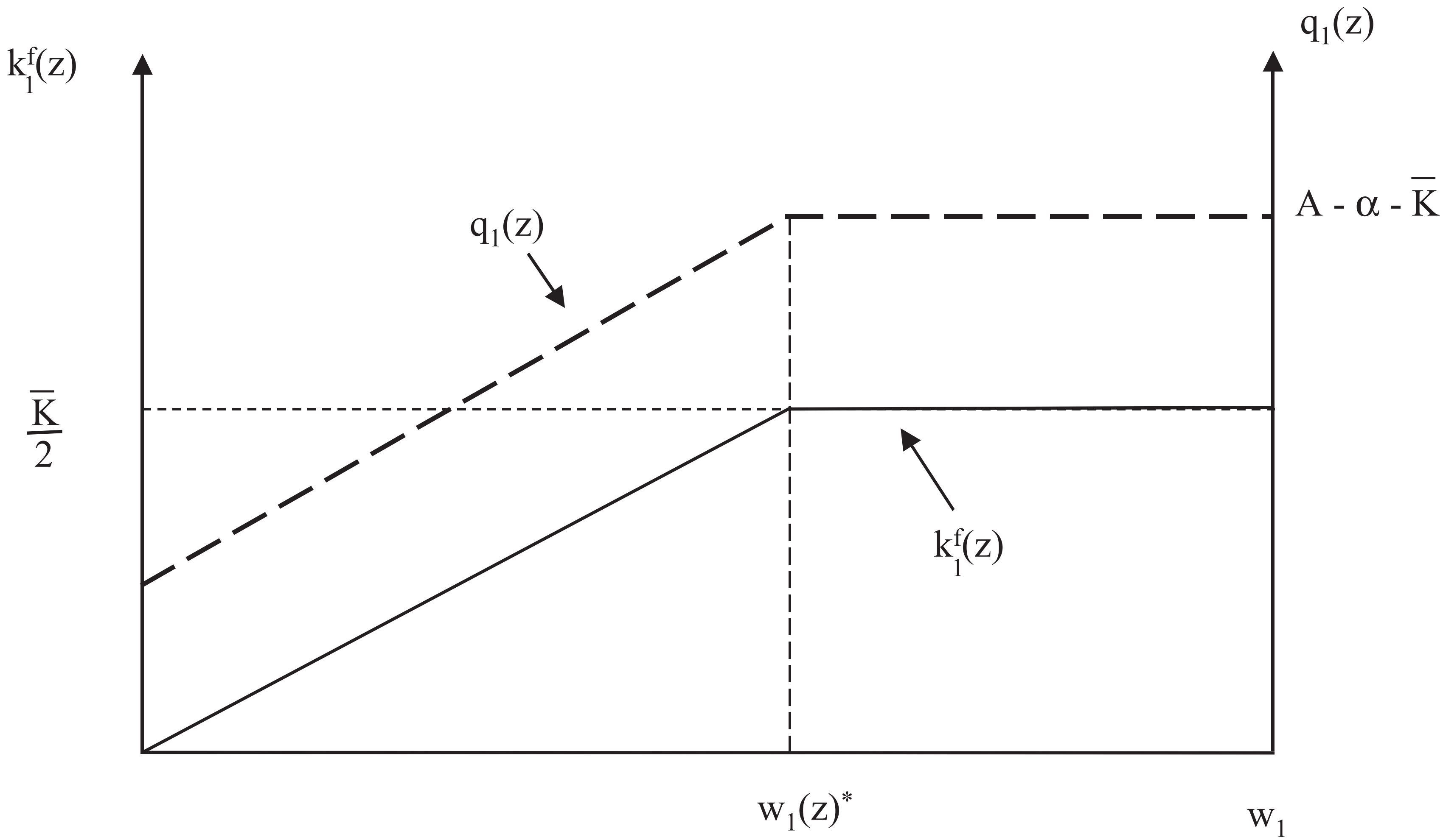


Fig. 1. Land values and farmer's wealth.

Figure 2

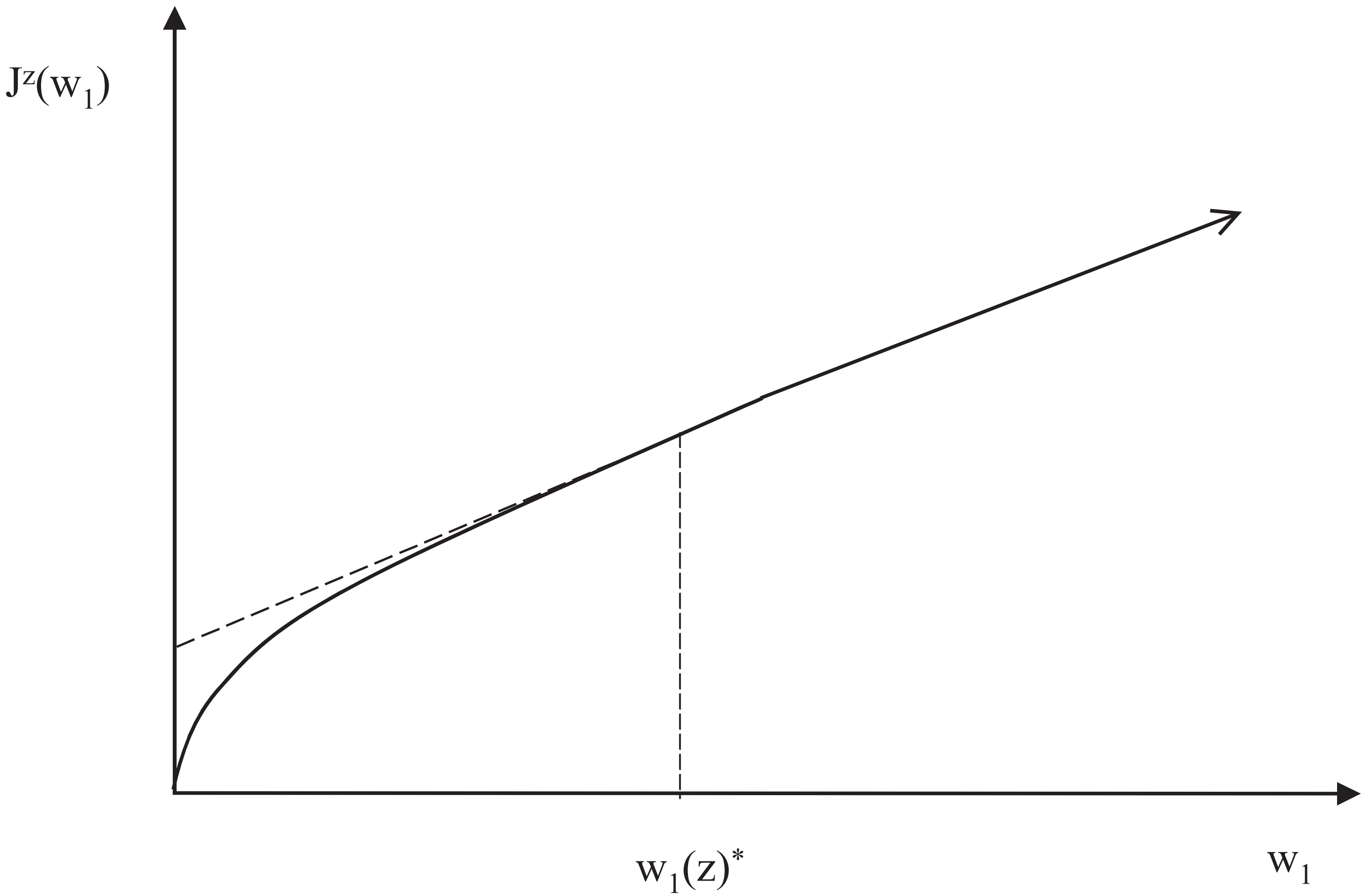


Fig. 2. Value function.

Figure 3

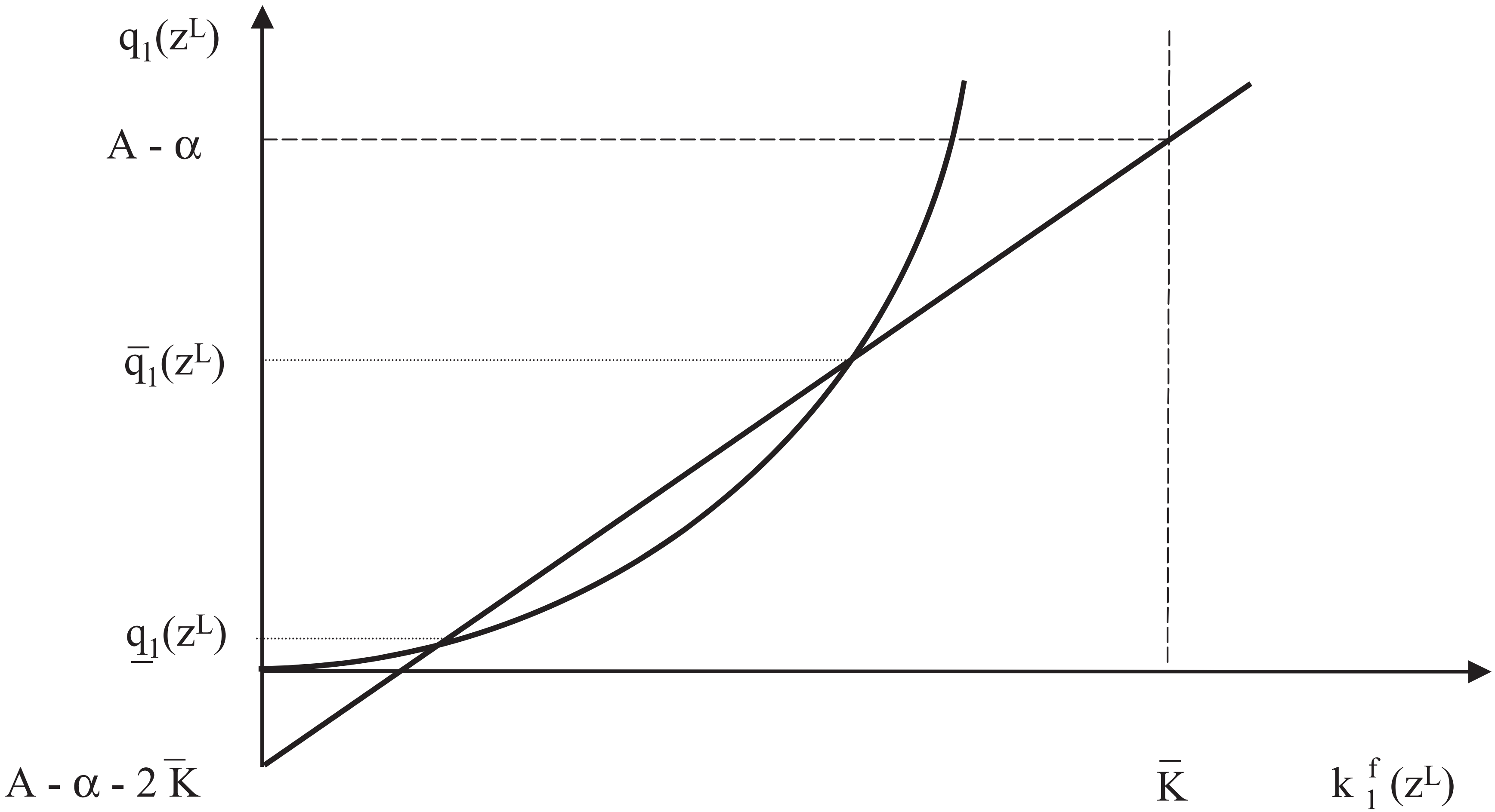


Fig. 3. Multiple equilibria.