Lecture:
After the Flood:
Financial Frictions & Central Banks

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Liquidity, Interbank Markets, & Central Banks

Allen and Gale (Ch. 6) & Allen, Carletti, & Gale (JME, 2009)
Boissay, Collard, and Smets (JPE, 2016)
Williamson (AER 2012 & JET, 2016)

Banks, Central Banks, and Financial Crises

Gertler and Karadi (JME 2011)
Gertler, Kiyotaki, and Queralto (JME 2012)
Farhi and Tirole (AER-PnP, 2009)

Banks, Central Banks, & Monetary Transmission

Goodfriend and McCallum (JME 2007)

INTRODUCTION

- FIs alter the characteristics of securities on the liability side of their balance sheets into securities with different characteristics on the asset side.
    1. This is the production function of FIs.
    2. Factor inputs are short term securities and information.
    3. Output is longer dated securities.
    4. $\Longrightarrow$ The asset maturity transformation process.

- This section of the course explores
    1. the interactions of market incompleteness, idiosyncratic risk, and aggregate risk on allocations,
    2. the role of interbank markets to allocate liquidity and risk,
    3. the place a central bank has in interbank markets,
    4. the meaning of a monetary policy when FIs and a central bank provide competing short term liabilities, and
    5. the role of monetary policy when short rates are near zero.

Introduction, I

- This section of Lecture 3 covers three papers.
- Allen & Gale (chapter 6) discuss that only when financial markets are not efficient is there are a role for regulators in stopping a "crisis" driven by aggregate shocks.
- Allen, Gale, & Carletti (JME, 2009) add
  1. idiosyncratic and aggregate shocks to alter the liquidity preferences of a FI depositors,
  2. an incomplete set of interbank markets $\Longrightarrow$ incomplete markets combined with disparate types of risk cause FIs to hoard liquidity,
  3. and a central bank (CB) that can improve allocations of resources across banks because of incomplete markets and insurable and non-insurable risk.
- Boissay, Collard, and Smets (JPE, 2016) wrap a RBC model around an interbank market subject to Holmström-Tirole style credit constraints that produce a demand for liquidity by financial intermediaries.

Introduction, II

- This section of also discusses Williamson (AER, 2012; JET, 2016).
- Williamson (AER, 2012) sees a central banks as a FI,
    1. but it has a monopolist's control of its liability $\Longrightarrow$ outside money,
    2. which competes with liquidity issued by FIs $\Longrightarrow$ inside money.
    3. In this environment, the issue is which restrictions generate equilibria in which public (outside) and private (inside) liquidity have or do not have positive value.
- Williamson (JET, 2016 and FRB-SL wp–008) incorporates
    1. long-term Treasury debt to study the impact of large scale purchases of these securities by a central bank
    2. on interest rates and equilibrium allocations when the value of an asset as collateral in trade is a function of its maturity.

A Model with Interbank Markets: Introduction, I

- Financial market efficiency is tied to market completeness in the banking models of Allen and Gale.
- For our purposes, complete markets exists only if Arrow-Debreu securities are available.
    1. Depositors do not buy AD securities $\Longrightarrow$ they receive payouts that are fixed rather than state contingent.
    2. If AD securities are not traded among FIs, a liquidity shock can force a FI to dump its assets at "fire sale" prices.
    3. $\Longrightarrow$ There is the potential for a run if depositors see (or expect) the FI (will) lack the resources to pay off early consumers in full.
    4. Aggregate liquidity shocks create the opportunity for a run to become systemic when several FIs need to pay off large numbers of early consumers simultaneously.
- Allen and Gale argue that financial market are inefficient when FIs cannot shift resources from states of world in which marginal utility is low to states in which it is high $\Longrightarrow$ risk sharing is incomplete.

A Model with Interbank Markets: Introduction, II

- Lets review the factors that lead to incomplete markets (but ignore markets can be completed by creating a sufficient volume of synthetic securities from underlying securities).
- Besides AD securities, several of these factors are
  1. a lack of perfect competition $\Longrightarrow$ prices do not adjust to clear markets,
  2. incomplete or asymmetric information $\Longrightarrow$ adverse selection and/or moral hazard,
  3. consumer welfare and the profits of firms are affected by the actions of other agents $\Longrightarrow$ externalities, and
  4. there are costs or limits to trading $\Longrightarrow$ markets are not frictionless and no trader has "deep pockets."
- Allen and Gale place great weight on the lack of AD securities.
- However, Krishnamurthy (JET, 2003) suggests the ability of AD securities to complete markets rests on the implicit assumption of perfectly elastic asset supply functions.

A Model with Interbank Markets: Set-Up

- A three period economy, $t = 0, 1, 2$, consisting of households and a 1-period risk-free liquid asset, and a risky 2-period asset.
- Households, who are risk averse, consume at date 1 or date 2,
  1. their endowment is a unit of the consumption good at $t = 0$,
  2. and share the preferences $\mathcal{V}(C_1, C_2) = \mathcal{U}(C_1) + \beta\mathcal{U}(C_2)$.
- There are S states of the world, $s = 1, 2, \ldots, S$, where the probability of s is $\pi_s > 0$, and $\sum_1^S \pi_s = 1$.
- The probability a household is an $t = 1$ consumer is $\lambda_i(s)$, i=A, B.
  1. $\lambda_i(s)$= the fraction of households in "region" i given the state s.
  2. At date 1, s and $\lambda_i(s)$ are revealed to agents in the economy.
- The risk-free asset returns a unit of the consumption good in state s tomorrow for every unit invested today.
- The long-dated asset needs a unit of the consumption good invested at $t = 0$ to return $1 + R(s)$ in state s at $t = 2$.

A MODEL WITH INTERBANK MARKETS: COMPLETE MARKETS

- Region i FIs only take deposits from region i households, $i = A, B$.
  1. Heterogeneity implies there are potential gains from trade.
  2. Since $\lambda_A(s) \gtreqless \lambda_B(s)$, A region FIs could trade with B region FIs to insure against idiosyncratic shocks.
  3. $\Longrightarrow$ An interbank market transfer liquidity across the regions.

- At $t=0$, deposit contracts offer i region households the payoff tuple $C_i = \left\{C_1(s, i), C_2(s, i)\right\}_{s=1}^{S}$ in exchange for their endowment.
  1. FIs face the ICC: $C_1(s, i) \leq C_2(s, i)$, $s = 1, 2, \ldots, S$ and $i = A, B$.
  2. The ICC guarantees that early consumers tell the truth about their type $\Longrightarrow$ late consumers cannot be better off by being dishonest about their type.

- Consumption is state dependent under the deposit contract.
  1. The state dependent prices of $C_1(s)$ and $C_2(s)$ are $p_1(s)$ and $p_2(s)$, $s = 1, 2, \ldots, S$.
  2. At $t=0$, FIs trade AD securities at $p_1(s)$ and $p_2(s)$ in perfectly competitive markets.

A MODEL WITH INTERBANK MARKETS: THE FI'S COMPLETE MARKETS PROBLEM

- Given complete markets and perfect competition, a region i FI maximizes household expected utility

$$\sum_{s=1}^{S} \pi_s \Big[\lambda_i(s)\mathcal{U}\Big(C_1(s,\, i)\Big) + (1-\lambda_i(s))\beta\mathcal{U}\Big(C_2(s,\, i)\Big)\Big],$$

by choosing C at $t=0$, subject to the ICC and the budget constraint

$$\sum_{s=1}^{S} \Big[\lambda_i(s)p_1(s)C_1(s,\, i)+(1-\lambda_i(s))p_2(s)C_2(s,\, i)\Big] \leq 1.$$

A Model with Interbank Markets: The FI's Complete Markets Problem

- Implicit in perfect competition is free entry of FIs $\Longrightarrow$ FI profits = 0.
- When FIs are long in the short asset, zero profits require $\sum_{s=1}^{S} p_1(s) = 1$.
- For the long asset, $\sum_{s=1}^{S} p_2(s)\big(1 + R(s)\big) \leq 1 \Longrightarrow p_1(s) \leq p_2(s)$.
- Otherwise, roll over the 1–period asset and never hold the 2–period asset.
- If FIs hold the 2–period asset, $\sum_{s=1}^{S} p_2(s)\big(1 + R(s)\big) = 1$ and $p_1(s) = p_2(s)$.

A Model with Interbank Markets: A Complete Markets Equilibrium

- When the ICC holds, $C_1(s, i) = C_2(s, i)$, $s = 1, 2, \ldots, S$ and $i = A, B$.
- Although complete risk sharing can exist in regions A and B, this is not necessarily true across the regions $\Longrightarrow$ not a first-best allocation.
- Characterize maximizing region A and B utility by choosing $C = \left\{C_A, C_B\right\}$, s.t. budget constraints when markets clear and the ICC holds.
- The allocation C is *attainable* given markets clear in $t = 0$, 1, and 2
  1. the amount of the consumption good invested in the 1– and 2–period assets sum to one,
  2. the demand for early consumption across regions A and B plus the amount the 1–period asset rolled over from $t=1$ into $t=2$ equals the payoff from investing in this asset at $t=0$, and
  3. the demand for late consumption across regions A and B equals the payoffs from the 1–period asset rolled over from $t=1$ into $t=2$ plus the return to investing in the 2–period asset at $t=0$.
- If no other allocation, call it $C^{\blacklozenge}$, exists that improves the ex ante welfare of regions A and B, C is *incentive efficient*.

A MODEL WITH INTERBANK MARKETS: INCOMPLETE CONTACTS & INCOMPLETE MARKETS

- Allan and Gale (chapter 6) associate incomplete contracts with non-state contingent deposit contracts.
  1. Let $\mathcal{D}$ be the payoff of a non-state contingent deposit for early consumers, but late consumers face ex ante state contingent payoffs $\Longrightarrow$ returns depend on the state at $t=2$ net of $\mathcal{D}$.
  2. A non-state contingent deposit contract imposes additional constraints on the set of attainable allocations.
  3. However, the non-state contingent deposit contracts are a reduced form for deeper structure that negate AD securities.
  4. If markets are complete and contracts are incomplete, equilibria can be *constrained efficient* $\Longrightarrow$ depositor welfare cannot be improved in the presence of these contracting frictions.

- Suppose there are liquidity preference shocks specific to regions A and B.
  1. If no markets exist to move liquidity between regions A and B, there is market incompleteness.
  2. Or the market exists, but the expected size of the shock, say, in region A is sufficiently large that ex ante FIs in region B are unwilling to trade.
  3. Ex ante they lack the resources to trade in this interbank market to fulfill their deposit contracts.

A Model with Interbank Markets: A FI Facing Incomplete Contacts & Markets

- The deposit contract offers $C_1 = \mathcal{D}$ and $C_2(s) = \dfrac{(1-\lambda_s\mathcal{D})(1+R)}{(1-\lambda_s)}$,
  1. where after paying a mass λ_s of early consumer $\mathcal{D}$ at t=1 a FI is left with $(1-\lambda_s\mathcal{D})(1+R(s))$ to return to a late consumer at t=1, and
  2. this payoff is scaled by the mass of these households, $1-\lambda_s$.

- Without an interbank market, a region i FI chooses $\mathcal{D}$ to maximize

$$\underline{\lambda}\mathcal{U}\big(\mathcal{D}\big) + \frac{\beta}{2}\left[(1-\lambda_H)\mathcal{U}\left(\frac{(1-\lambda_H\mathcal{D})(1+R)}{(1-\lambda_H)}\right) + (1-\lambda_L)\mathcal{U}\left(\frac{(1-\lambda_L\mathcal{D})(1+R)}{(1-\lambda_L)}\right)\right],$$

where the ICC, $\mathcal{D} \leq C_2(s)$, is assumed to hold, $\underline{\lambda}$ is the mass of early consuming households in region i, and $s = L, H$ denoting low and high liquidity demand.

- Optimality requires a FI to match an increase in early consumers, say from $\underline{\lambda}$ to $\overline{\lambda}$, with an increase in the expected $\mathcal{U}'(\cdot)$ of the late consumers

$$\overline{\lambda}\mathcal{U}'\big(\mathcal{D}\big) = \beta(1+R)\left[\frac{\lambda_H}{2}\mathcal{U}'\left(\frac{(1-\lambda_H\mathcal{D})(1+R)}{(1-\lambda_H)}\right) + \frac{\lambda_L}{2}\mathcal{U}'\left(\frac{(1-\lambda_L\mathcal{D})(1+R)}{(1-\lambda_L)}\right)\right],$$

- The FI accomplishes this task by selling some of the 2–period asset it owns, which raises $\mathcal{U}'\big(C_2(s)\big)$, $C_2(s)$ falls $\Longrightarrow$ there is no interbank market to hedge this risk.

Introduction: Allen, Carletti, and Gale (JME, 2009)

- Suppose an interbank market exists in which region A and B FIs trade a AD security to insure against large liquidity shocks.
- ACG show that an interbank market helps FIs to *reallocate* liquidity to those facing an unexpected increase in early consumers.
- Liquidity preference is driven by household-specific and aggregate shocks.
  1. Assume idiosyncratic and aggregate uncertainty is insufficient to generate bank runs $\Longrightarrow$ the aggregate liquidity shock is too small to soak up all the liquidity (*i.e.* short asset) owned by a FI.
  2. FIs alter their liquidity in response to changes in asset prices $\Longrightarrow$ adjust to clear markets and satisfy optimality conditions.
- Incomplete markets create an opening for a "central bank" to engage in actions that produce the *constrained efficient* equilibrium.
  1. The CB fills the role of the social planner, according to ACG.
  2. The constrained efficient equilibrium results from the CB trading short and long assets with FIs $\Longrightarrow$ OMOs, which move rates creating incentives for FIs to hold short and long assets.
  3. Still, FIs may stop trading in the interbank market backed by a CB if they anticipate rising demand for liquidity.

THE ACG MODEL: LIQUIDITY PREFERENCES AND BUDGET CONSTRAINTS

- The set-up is similar to Allen and Gale (chapter 6) with two exceptions.
- A household is an early consumer with probability λ
  1. $\lambda_{\theta,i} = \alpha_i + \epsilon\,\theta$, which is private information, where $i = H, L$,
  2. H (L) = an early consumer in need of high (low) liquidity.
  3. α is an idiosyncratic shock, $\alpha_H = \overline{\alpha} + \eta$, $\alpha_L = \overline{\alpha} - \eta$, $\Pr(\alpha_i) = 0.5$, $\eta > 0$, and $0 < \alpha_L \leq \alpha_H < 1$, and
  4. the aggregate shock $\theta = [0, 1]$, $\Pr(\theta = 0) = \pi$, $\Pr(\theta = 1) = 1 - \pi$.
  5. Its scale is $0 < \epsilon \leq \eta \Longrightarrow$ otherwise aggregate uncertainty dominates idiosyncratic uncertainty causing the interbank market to shut down.
  6. Household preferences are $\lambda_{\theta,i}\mathcal{U}\big(C_1\big) + (1 - \lambda_{\theta,i})\mathcal{U}\big(C_2\big)$.
  7. At t=0, the endowment is one unit of the single consumption good and zero during $t = 1, 2$.
- FIs operate in perfectly competitive markets $\Longrightarrow$ free entry.
  1. The 1– and 2–period assets have non-stochastic returns paying one unit of the consumption good and $1 + R$ units.
  2. Households are promised $\mathcal{D}$ if they are early consumers.
  3. FIs put $\mathcal{Y}$ $(1 - \mathcal{Y})$ of their portfolio in the short (long) asset.
  4. ICC imposes $\lambda\mathcal{D} + (1 - \mathcal{Y})\mathcal{D}P_\theta \big/ (1 + R) \leq \mathcal{Y} + (1 - \mathcal{Y})P_\theta$ $\Longrightarrow$ since $C_1 \leq C_2$, where P_θ is the price of the 2–period asset.

The ACG Model: The Planner's Problem

- This is a planner's problem, which serves as a benchmark, is

$$\text{Max}_{\{\mathcal{Y},\, \mathcal{D}\}} \Big\{ \pi\Big[\lambda_0 \mathcal{U}\big(\mathcal{D}\big) + (1-\lambda_0)\mathcal{U}\big(C_{2,0}\big)\Big] + (1-\pi)\Big[\lambda_1 \mathcal{U}\big(\mathcal{D}\big) + (1-\lambda_1)\mathcal{U}\big(C_{2,1}\big)\Big]\Big\},$$

s.t. $\lambda_0\mathcal{D} \le \mathcal{Y}$, $(1-\lambda_0)C_{2,0} = \mathcal{Y} - \lambda_0\mathcal{D} + (1-\mathcal{Y})(1+R)$, $\lambda_1\mathcal{D} \le \mathcal{Y}$, $(1-\lambda_1)C_{2,1} = \mathcal{Y} - \lambda_1\mathcal{D} + (1-\mathcal{Y})(1+R)$, $0 \le \mathcal{D}$, and $\mathcal{Y} \in (0, 1)$, where

1. $\lambda_0 = \overline{\alpha}$ when $\theta = 0$, $\lambda_1 = \overline{\alpha} + \epsilon$ when $\theta = 1$,
2. $C_{2,0}$ is late consumption when $\theta = 0$,
3. $C_{2,1}$ is late consumption when $\theta = 1$,

- Late consumption is $(1-\lambda_s)C_{2,s}$ in state s, which is less than or equal to
  1. resources not eaten by early consumers in state s (rolled over from t=1 to t=2 in the 1-period asset), $\mathcal{Y} - \lambda_s\mathcal{D}$, plus the return to the 2-period asset, $(1-\mathcal{Y})(1+R)$.
  2. The SP ignores idiosyncratic risk in λ_s because it is diversified away.

The ACG Model: A Constrained Efficient Allocation

- Optimality is $\lambda_1 \mathcal{D}^* = \mathcal{Y}^*$.
  1. Suppose not by fixing $\mathcal{D}^*$ and lower $\mathcal{Y}^*$ to raise the expected utility
  2. of a depositor using the fact $0 < R \Longrightarrow \lambda_0 \mathcal{D}^* < \mathcal{Y}^*$ because $\lambda_0 < \lambda_1$.

- If the aggregate shock to liquidity preferences is not realized, $\theta = 0$,
  1. more of the 1-period asset is rolled over from t=1 to t=2
  2. $\Longrightarrow$ extra resources for late consumers.

THE ACG MODEL: OPTIMALITY OF THE CONSTRAINED EFFICIENT ALLOCATION

- The constrained efficient allocation sets $\lambda_1\mathcal{D}^* = \mathcal{Y}^*$, which in turn gives $C_{2,0} = \dfrac{(\lambda_1 - \lambda_0)\mathcal{D} + (1 - \lambda_1\mathcal{D}^*)(1+R)}{1-\lambda_0} = \dfrac{\epsilon\mathcal{D} + (1 - \lambda_1\mathcal{D}^*)(1+R)}{1-\lambda_0}$, where $\lambda_1 - \lambda_0 = \epsilon$ and $C_{2,1} = \dfrac{(1 - \lambda_1\mathcal{D}^*)(1+R)}{1-\lambda_1}$.
- Use the constrained efficient allocation to write the planner's problem as

$$\text{Max}_{\mathcal{D}} \left\{\pi\left[\lambda_0\mathcal{U}\big(\mathcal{D}\big) + (1-\lambda_0)\mathcal{U}\left(\frac{\epsilon\mathcal{D} + (1-\lambda_1\mathcal{D})(1+R)}{1-\lambda_0}\right)\right] + (1-\pi)\left[\lambda_1\mathcal{U}\big(\mathcal{D}\big) + (1-\lambda_1)\mathcal{U}\left(\frac{(1-\lambda_1\mathcal{D})(1+R)}{1-\lambda_1}\right)\right]\right\}.$$

- At the constrained efficient allocation, the FONC is

$$\pi\left[\lambda_0\mathcal{U}'\left(\mathcal{D}^*\right) + (1-\lambda_0)\Big[\lambda_1(1+R) - \epsilon\Big]\mathcal{U}'\left(\frac{\epsilon\mathcal{D}^* + (1-\lambda_1\mathcal{D}^*)(1+R)}{1-\lambda_0}\right)\right] = (1-\pi)\left[\lambda_1\mathcal{U}'\left(\mathcal{D}^*\right) + (1-\lambda_1)\lambda_1(1+R)\mathcal{U}'\left(\frac{(1-\lambda_1\mathcal{D}^*)(1+R)}{1-\lambda_1}\right)\right].$$

- $C_{2,1} < C_{2,0}$ per capita, but the latter's "expected return" on the 2–period asset is reduced by ϵ to provide complete risk sharing across the states $\theta = 0$ and $\theta = 1$.

THE ACG MODEL: ADD AN INTERBANK MARKET

- Imagine there is a unit of FIs taking addresses on the unit interval.
- At $t=0$, FIs trade the 2-period asset in an interbank market at price P_θ $\Longrightarrow$ this security is state contingent.
- FIs use the interbank market to adjust their portfolios after observing idiosyncratic and aggregate liquidity shocks at $t=1$.
  1. At $t=1$, FIs with excess liquidity buy the 2-period asset from FIs experiencing an unexpected volume of withdrawals $\Longrightarrow$ the long asset is state contingent.
  2. The prices of 1- and 2-period assets equal one at $t=0$ $\Longrightarrow$ otherwise, ex ante, FIs would not trade because the return on the 1-period asset $< 1 + R$.
- For there to be no bank runs, incentive compatibility demands that $\mathcal{Y}$ and $\mathcal{D}$ are chosen to satisfy $C_{2,\theta,i} = \left[1 - \mathcal{Y} + \frac{\mathcal{Y} - \lambda_{\theta,i}\mathcal{D}}{P_\theta}\right]\left[\frac{1+R}{1-\lambda_{\theta,i}}\right]$, $\theta = 0, 1$ and $i = H, L$.
  1. $1 - \mathcal{Y} + (\mathcal{Y} - \lambda_{\theta,i}\mathcal{D})\Big/P_\theta$ = 2-period asset bought at $t=0$ + value of this asset purchased during $t=1$ $\Longrightarrow$ FIs are selling the 2-period asset if the latter is < 0.
  2. $(1+R)\Big/(1-\lambda_{\theta,i})$ = 2-period asset's return scaled by mass of late consumers.

THE ACG MODEL: THE INTERBANK MARKET'S IMPACT ON THE FI'S PROBLEM

- At $t=0$, a FI chooses $\{\mathcal{Y}, \mathcal{D}\}$ to maximize

$$\frac{1}{2}\Big\{\pi\left[\lambda_{0,H}\mathcal{U}\big(\mathcal{D}\big) + (1-\lambda_{0,H})\mathcal{U}\big(C_{2,0,H}\big) + \lambda_{0,L}\mathcal{U}\big(\mathcal{D}\big) + (1-\lambda_{0,L})\mathcal{U}\big(C_{2,0,L}\big)\right]$$

$$+ \ (1-\pi)\left[\lambda_{1,H}\mathcal{U}\big(\mathcal{D}\big) + (1-\lambda_{1,H})\mathcal{U}\big(C_{2,1,H}\big) + \lambda_{1,L}\mathcal{U}\big(\mathcal{D}\big) + (1-\lambda_{1,L})\mathcal{U}\big(C_{2,1,L}\big)\right]\Big\},$$

s.t. $0 \le \mathcal{D}$, $\mathcal{Y} \in (0, 1)$, and P_θ, $\theta = 0, 1$, is taken as given.

- With respect to $\mathcal{Y}$ and $\mathcal{D}$, the FONCs are

$$\pi\left(\frac{P_0 - 1}{P_0}\right)\left[\mathcal{U}'\left(C_{2,0,H}\right) + \mathcal{U}'\left(C_{2,0,L}\right)\right] = (1-\pi)\left(\frac{1-P_1}{P_1}\right)\left[\mathcal{U}'\left(C_{2,1,H}\right) + \mathcal{U}'\left(C_{2,1,L}\right)\right],$$

and

$$\left[\overline{\alpha} + (1-\pi)\epsilon\right]\mathcal{U}'\left(\mathcal{D}\right) = \frac{1+R}{2}\left[\frac{\alpha_H\mathcal{U}'\left(C_{2,0,H}\right) + \alpha_L\mathcal{U}'\left(C_{2,0,L}\right)}{P_0}\right.$$

$$\left. + \ \frac{(\alpha_H + \epsilon)\mathcal{U}'\left(C_{2,1,H}\right) + (\alpha_L + \epsilon)\mathcal{U}'\left(C_{2,1,L}\right)}{P_1}\right].$$

The ACG Model: Optimality and the Interbank Market

- The FONC w/r/t $\mathcal{Y}$ represents risk sharing for late consumers across aggregate states $\theta = 0$ and $\theta = 1$.

- Risk sharing for early and late consumers across the idiosyncratic and aggregate shocks is described by the FONC w/r/t $\mathcal{D}$.

- Ex ante FIs engage in risk sharing by allocating the 1–period asset
  1. to the half of the market subject to unanticipated idiosyncratic demand from early consumers from other half that is not.
  2. Still FIs act against an aggregate liquidity shock $\theta = 1$.
  3. $\Longrightarrow \mathcal{Y} \geq \lambda_1 \mathcal{D} = (\overline{\alpha} + \epsilon)\mathcal{D} \Longrightarrow \mathcal{Y} > \lambda_0 \mathcal{D} = \overline{\alpha}\mathcal{D}$, which
  4. implies that FIs hold more liquidity than is optimal (compared with the constrained efficient allocation) in state $\theta = 0$.
  5. Arbitrage requires $P_0 = 1 + R$; otherwise $P_0 > 1 + R$ face FIs with an incentive to hold only the 1–period asset and only the 2–period asset given $P_0 < 1 + R$.

- As for the constrained efficient allocation, the interbank market equilibrium sets $\mathcal{Y} = \lambda_1 \mathcal{D} = (\overline{\alpha} + \epsilon)\mathcal{D}$.
  1. If $P_1 = 1 + R$, contradicts the equilibrium price $P_0 = 1 + R$ $\Longrightarrow$ FIs would not hold the 1–period asset.
  2. Substitute $(\overline{\alpha} + \epsilon)\mathcal{D} = \mathcal{Y}$ and $P_0 = 1 + R$ into the FONCs to solve for P_1 and $\mathcal{D}$.

The ACG Model: A Central Bank and an Interbank Market

- The central bank (CB) engages in OMOs in the interbank market.
- ACG show a CB's OMOs achieves the constrained efficient allocation, which is feasible $\Longrightarrow$ optimal allocations for households.
    1. The OMOs set the prices of the 1- and/or 2-period assets.
    2. The CB does not issue its own liability either real or nominal.
    3. $\Longrightarrow$ The relative price of the CB's liability to a private liability is not the policy tool.
- Policy experiments add or subtract liquidity from the interbank market
    1. to set asset prices consistent with the constrained efficient allocation.
    2. $\Longrightarrow$ Change the maturity structure of the asset side of FI balance sheets.
- From whom does the CB obtain the resources to conduct OMOs?

THE ACG MODEL: ONLY IDIOSYNCRATIC LIQUIDITY RISK

- Given $\eta > 0$ and $\epsilon = 0 \Longrightarrow \frac{1}{2}$ of the FIs see $\lambda_H = \overline{\alpha} + \eta$ and the rest $\lambda_L = \overline{\alpha} - \eta$.
- A fiscal authority levies a lump sum tax of $\mathcal{X}_0$ on households at t=0.
  1. $\Longrightarrow$ Households post-tax endowment is $1 - \mathcal{X}_0$.
  2. At t=0, FIs hold $\mathcal{Y} - \mathcal{X}_0$ in the short asset and $1 - \mathcal{Y}$ in the long asset.
  3. The tax revenue is given to the CB to conduct OMOs.
- A FI needs $\mathcal{Y} - \mathcal{X}_0 - \lambda_i\mathcal{D}$ in liquidity at t=0 $\Longrightarrow$ buy or sell the short asset.
  1. The CB buys $\mathcal{X}_0$ of the long asset in the interbank market driving $P = 1$ $\Longrightarrow$ insufficient (excess) liquidity signaled by $P > (<)\ 1$ and $X_0 > (<)\ 0$.
  2. At t=0, $0.5(\mathcal{Y}^* - \mathcal{X}_0 - \lambda_H\mathcal{D}^*) + 0.5(\mathcal{Y}^* - \mathcal{X}_0 - \lambda_L\mathcal{D}^*) + \mathcal{X}_0 = 0$ clears the interbank market $\Longrightarrow \mathcal{Y}^* = \overline{\alpha}\mathcal{D}^*$.
- A FI has $1-\mathcal{X}_0-\lambda_i\mathcal{D}$ in the long asset $\Longrightarrow$ at t=2 total payments to late consumers are
  1. $\beta_{2,i} = \dfrac{(1 - \mathcal{X}_0 - \lambda_i\mathcal{D})(1 + R)}{(1 - \lambda_i)} \Longrightarrow \beta_{2,H} + \beta_{2,L} = \mathcal{D}^*(1 + R) \equiv \beta_2$, if $X_0^* = 1 - \mathcal{D}^*$.
  2. Remember the CB owns $\mathcal{X}_0$ of the 2-period asset $\Longrightarrow$ its portfolio income is $\mathcal{X}_0(1 + R)$, which is rebated to the mass of $1 - \lambda$ of all late consumers.
  3. Call these rebates $\gamma_2^* = \dfrac{\mathcal{X}_0^*(1 + R)}{1 - \lambda} \Longrightarrow$ add to $\beta_2 \Longrightarrow C_2^* = \dfrac{(1 - \mathcal{Y}^*)(1 + R)}{1 - \lambda}$.
- The tax and OMO scheme annihilates the impact of the idiosyncratic shocks on the FIs balance sheet $\Longrightarrow$ satisfy ICC.

THE ACG MODEL: ONLY AGGREGATE LIQUIDITY RISK

- Given $\eta = 0$ and $\epsilon > 0$, FIs face $\lambda_0 = \overline{\alpha}$ and $\lambda_1 = \overline{\alpha} + \epsilon \Longrightarrow$ SP's optimal solution sets

$$\mathcal{Y}^* = \lambda_1 \mathcal{D}^*, \quad C^*_{2,0} = \frac{\epsilon\mathcal{D} + (1 - \lambda_1\mathcal{D}^*)(1+R)}{1-\lambda_0}, \text{ and } C^*_{2,1} = \frac{(1 - \lambda_1\mathcal{D}^*)(1+R)}{1-\lambda_1}.$$

- Suppose that the aggregate state is $\theta = 0 \Longrightarrow$ there is excess liquidity of $\epsilon\mathcal{D}^*$.
  1. FIs exchange $\epsilon\mathcal{D}$ of the short asset for CB debt in the amount X_1 at $t=1$ $\Longrightarrow$ removes liquidity from the interbank market.
  2. The CB's debt pays $1 + R$ at $t=2 \Longrightarrow$ lowers the price of the long asset to $P_0 = 1$
  3. The fiscal authority raises revenue, $\gamma_{2,0}$, to finance this government debt by taxing late consumers $\Longrightarrow$ the CB is not independent of the fiscal policymaker.

- In state $\theta = 0$ at $t=2$, late consumers receive $\beta^*_{2,0} = \dfrac{(1 - \mathcal{Y}^* - \epsilon\mathcal{D}^*)(1+R)}{1-\lambda_0}$ from FIs, but pay a lump sum tax $\gamma^*_{2,0} = \dfrac{\epsilon\mathcal{D}^* R}{1-\lambda_0}$ to the fiscal authority $\Longrightarrow C^*_{2,0} = \beta^*_{2,0} + \gamma^*_{2,0}$.

- The CB takes no policy actions in state $\theta = 1$ because the interbank market allocates liquidity optimally, according to the constrained efficient allocation.

The ACG Model: Idiosyncratic and Aggregate Liquidity Risk

- Assuming $\eta > 0$ and $\epsilon > 0$, the fiscal authority and the CB mix
  1. the optimal tax and OMO schemes constructed under only idiosyncratic risk and only aggregate risk.
  2. This is feasible because the optimal tax and OMO schemes are linear in $\mathcal{Y}^*$ and $\mathcal{D}^*$.

- The fiscal authority and CB achieve the constrained efficient allocations in the face of idiosyncratic and aggregate risk
  1. because the interbank market is *incomplete*.
  2. FIs are unable on their own to internalize the idiosyncratic and aggregate liquidity preference risk of their depositors.

- Deposit contracts are also incomplete $\Longrightarrow$ not state contingent.
  1. A SP replicates the constrained efficient allocations in this case.
  2. The CB plays the same role as the SP in the interbank market $\Longrightarrow$ supporting the CB's policy actions is the fiscal authority.

Credit Constraints & Liquidity in a DSGE Model

- Boissay, Collard, and Smets (JPE, 2016) build a RBC model with an interbank market grounded in notions of credit constraints and liquidity associated with Holmström and Tirole.

- Banks differ according to their productivity, p_t, at intermediation, which is private information $\Longrightarrow$ CDF $\mu(\cdot)$ of p_t is known, $p_t \sim IID$, and unknown.
    1. Heterogeneity creates demand for liquidity by FIs more efficient at transforming short-term liabilities into long-term assets.
    2. Funds supplied by less efficient FIs $\Longrightarrow$ an interest rate wedge reducing activity because more productivity FIs gain funds to create assets.

- The interbank market is competitive and its outcome are observable publicly as modeled by Boissay, Collard, and Smets (BCS).
    1. The most productive FIs wants to borrow an infinite amount.
    2. $\Longrightarrow$ Efficient and first best for this FI to corner the interbank market.

Asymmetric Information and Moral Hazard

- Along with asymmetric information, assume moral hazard exists to obtain an interior solution in the interbank market.
- The moral hazard occurs because FIs can lend to firms or invest in an "outside" option with a certain return that is less than the return on loans.
  1. Investment in the outside option is illiquid $\Longrightarrow$ source of liquidity shortages because FIs cannot close out this asset to lend to firms.
  2. BCS assume FIs can make off with a fraction of the outside option without consequences $\Longrightarrow$ this investment is private information.
  3. A FI lending in the interbank cannot recover funds lost to a FI that has run off with returns to the outside option $\Longrightarrow$ ex post moral hazard.
  4. This is akin to the HT problem of pledging returns to lender.
  5. FIs lending in the interbank market need borrowers to put their own funds in loans $\Longrightarrow$ leverage contains trade-offs for FIs.
  6. The trade-off is encapsulated in the ICC $\Longrightarrow$ bounds return to the outside option, γ, by loan rate in the interbank market for a FI that decamps.
  7. The interbank market has an externality $\Longrightarrow$ created by the marginal FI moving between the demand (negative) and supply (positive) sides.

The Interbank Market in a DSGE Model

- BCS embed the interbank market in a one-sector RBC model.
- Households own FIs, but not firms $\Longrightarrow$ firms have to obtain funds to accumulate capital from FIs.
- A liquid interbank market relies on aggregate household saving, $A_t \leq \overline{A}_t$,
  1. $\overline{A}_t \equiv \mathrm{MP}_k^{-1}\left(\dfrac{\overline{R}_K + \delta - 1}{Z_t}\right) \Longrightarrow$ the threshold demand for capital is a function of the threshold return to capital and TFP, where $\delta \in (0,\, 1)$.
  2. Aggregate loan supply $\ell_t = A_t \leq \overline{A}_t$, otherwise $\ell_t = \left[1 - \mu\left(\dfrac{\gamma}{R_{K,t}}\right)\right] A_t$ $\Longrightarrow$ FIs not lending have $p_t < \overline{p}_t = \gamma / R_{K,t}$, which signals a liquidity crunch.
  3. However, $R_{K,t} < \overline{R}_K$ does not generate a liquidity crunch, but TFP can.
  4. The TFP threshold $\overline{Z}_t \equiv \dfrac{\overline{R}_K + \delta - 1}{\mathrm{MP}_K(A_t)} \leq Z_t$ is necessary for there to be sufficient interbank liquidity for FIs to lend to firms $\Longrightarrow$ probability of a liquidity crunch depends on probability of $Z_t < \overline{Z}_t$ conditional on A_{t-1} and Z_{t-1}.
  5. BCS study liquidity crises in an interbank market, not insolvency crises for FIs.

Introduction: Williamson (AER 2012)

- Liquidity takes many forms.
  1. **Private liquidity**: a deposit at a FI, which is its liability, but
  2. most any liability a FI issues has potential to provide liquidity services.
  3. **Public liquidity**: a CB's liabilities and government securities.
- Since (before) Friedman's and Schwartz's A Monetary History of the United States: 1867–1960 (1963), monetary policy is about
  1. a CB managing the relative price of its liability to a short term asset issued by private agents.
  2. $\Longrightarrow$ Control inflation and negate its impact on the real economy.
- Williamson builds a model in which private and public liabilities can be close substitutes $\Longrightarrow$ describe restrictions on economic primitives
  1. under which private, public, or private and public liabilities circulate.
  2. $\Longrightarrow$ Mediums of exchange that are stores of positive and finite value.
- If liquidity services are extracted from private and Treasury securities, do the agents supplying these assets need to be modeled?
- Williamson argues the state of fiscal policy limits a CB's actions.
  1. These actions are also s.t. the "costs" of the CB's operating mechanism.
  2. $\Longrightarrow$ are there sufficiently liquid assets to substitute for a CB's liability?

Williamson (AER 2012): Sketch of the Baseline Model

- Baseline model starts from
  1. Lagos and Wright (2005, "A Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy* 113, 463–484).
  2. Lagos and Rocheteau (2008, "Money and Capital as Competing Media of Exchange," *Journal of Economic Theory* 142, 247–258).

- Analysis relies on households with quasi-linear preferences afflicted by uncertainty about their intertemporal liquidity preferences.

- Public assets include fiat currency and nominal Treasury bonds.
  1. There is a role for Diamond-Dybvig style FIs, which offer liquidity insurance to their depositors.
  2. FIs trade to diversify liquidity risk $\Longrightarrow$ endogenous asset returns.

- With only public liquidity, there are four potential equilibria.
  1. Sufficient supply of liquid assets.
  2. Insufficient supply of liquid assets.
  3. "A liquidity trap."
  4. Friedman rule.

Williamson (AER 2012): Multiple Equilibria

- Sufficient supply of liquid assets: OMOs only shift the price level.
- Insufficient supply of liquid assets: OMOs are non-neutral.
  1. An *illiquid* interbank market is the source of monetary non-neutrality.
  2. Hoard liquidity because the household's rate of time preference > than the real rate $\Longrightarrow$ liquidity premium on Treasuries.
  3. An OMO purchase of Treasuries removes liquidity from the market place $\Longrightarrow$ price level rises but real rate falls.
- Williamson defines a liquidity trap as occurring
  1. "if total liquid assets (public and private) are sufficiently scarce, and currency is sufficiently plentiful relative to other assets." (p. 2572)
  2. In a liquidity trap, the nominal rate = 0, OMOs alter neither the price level nor real rates $\Longrightarrow$ FIs hold the additional cash.
- The Friedman rule is not a liquidity trap according to Williamson.
  1. Instead, equate returns on all assets to the rate of time preference.
  2. Interbank trading is efficient $\Longrightarrow$ FI demand for assets perfectly elastic.

Williamson (AER 2012): Summarize Results

- Optimal monetary policy faces an inflation trade-off
    1. Inflation is a tax when transacting in fiat currency and liquid assets.
    2. Trading in fiat currency sometimes improves welfare but not always.
    3. Welfare is improved when liquid assets are a medium of exchange in Lagos-Wright models.

- Williamson points out LSAPs/QE policies are aimed at the wrong problem.

- During the recent financial crisis and recession, the problem was liquid assets were in scarce supply.
    1. Rather than buying scarce assets, which lowers the real rate,
    2. monetary policy should raise the real rate with OMOs that sell Treasuries into the market.
    3. Perhaps, the problem is drawing parallels with the Great Depression?
    4. There was excess demand for currency during the GP suggesting OMOs that buy Treasuries would have been the correct policy.

WILLIAMSON (AER 2012): MARKET STRUCTURE OF THE BASELINE MODEL

- Discrete time: $t = 0, 1, 2, \ldots$, but t has separate early and late subperiods.
  1. A centralized market (CM) opens in the first subperiod.
  2. A decentralized market (DM) opens in the second subperiod.
- Households are either "buyers" or "sellers."
  1. There is a unit mass of buyers and a unit mass of sellers.
  2. Buyers (Sellers) can produce goods only in the CM (DM).
  3. $\Longrightarrow$ FIs improve welfare by moving resources across the subperiods.
- The CM is a conventional Walrasian market cleared by price adjustment.
  1. Buyers (Bs), sellers (Ss), FIs, and monetary and fiscal policymakers trade in the CM.
  2. Subsequent to the CM closing, whether a B will be monitored or not when trading with a S in the DM of $t+1$ is revealed.
- Bs and Ss are matched randomly in the DM.
  1. S's history is private and B can "default" $\Longrightarrow$ S will not accept B's debt.
  2. Assume that a fraction ρ of trades in the DM use only fiat currency $\Longrightarrow$ S only accepts cash.
  3. $\Longrightarrow$ W/o monitoring B has no credible medium of exchange to offer.
  4. FIs monitor the $1 - \rho$ of remaining trades in the DM $\Longrightarrow$ Bs use FIs to transfer ownership of goods to Ss at zero cost.

WILLIAMSON (AER 2012): PREFERENCES AND TECHNOLOGY

- A B has preferences $\mathbf{E}_t \left\{ \sum_{j=0}^{\infty} \beta^j \left[-\mathcal{H}_{t+j} + \mathcal{U}(x_{t+j}) \right] \right\}$, $\beta \in (0, 1)$,
  1. where $\mathcal{H}_{t+j}$ and x_t are labor supply net of consumption in the CM and consumption in the DM,
  2. $\mathcal{U}(0) = 0$, $\mathcal{U}'(0) = \infty$, $\mathcal{U}(\infty) = 0$, $-x\dfrac{\mathcal{U}''(x)}{\mathcal{U}'(x)} < 1 \;\; \forall \; x \geq 0$,
  3. given $\hat{x} > 0$, $\mathcal{U}(\hat{x}) - \hat{x} = 0$, where $\hat{x}$ is defined by $\mathcal{U}'(\hat{x}) = 1$,
  4. and optimal x is labeled x^*.

- A S has preferences $\mathbf{E}_t \left\{ \sum_{j=0}^{\infty} \beta^j \left[X_{t+j} - h_{t+j} \right] \right\}$, where X_t is consumption in the CM and h_t is labor supply in the DM.

- Bs and Ss have access to a technology that transmutes a unit of labor into a unit of the nonstorable consumption good of the economy.

Williamson (AER 2012): The Government

- At $t=0$, the government budget constraint is $\phi_0\left(M_0 + B_0\right) + \tau_0 = 0$, where
  1. ϕ_0 is the initial "relative" price of cash per unit of the consumption good $\Longrightarrow$ purchasing power of money (*i.e.*, inverse of the price level),
  2. M_0, B_0, and τ_0 are the initial stock of fiat currency, one-period nominal bonds, and a lump sum tax.
  3. The government has only assets, no liabilities, at $t=0 \Longrightarrow$ Bs, Ss, and FIs do not have initial endowments of cash and bonds.

- Bs and Ss buy M_t and B_t from the government at price ϕ_t when the CM is open during t, while Bs pay τ_t prior to this market opening.

- When the CM opens during t, the government pays q_t units of cash per unit of nominal bond to Bs and Ss carrying this outside asset over from $t-1$ to t.

- Thus, the government budget constraint is

$$\phi_t\left(M_t + B_t\right) + \tau_t = \phi_t\left(M_{t-1} + q_t B_{t-1}\right), \quad t = 1, 2, \ldots, \infty.$$

Williamson (AER 2012): Equilibria and Passive Fiscal Policy

- The equilibria of the benchmark model rely on monetary policy taking the "lead" in the policy game.
  1. Monetary policy takes actions without considering fiscal policy.
  2. Fiscal policy closes the government's budget constraint $\Longrightarrow$ adjust τ_t to satisfy the government's budget constraint given $\{M_t, B_t\}_{t=1}^{\infty}$.

- Define $r_{t+1} \equiv \dfrac{\phi_{t+1} q_{t+1}}{\phi_t}$ $\Longrightarrow$ gross real rate on government debt, where $\dfrac{\phi_{t+1}}{\phi_t}$ is the real return on cash (*i.e.*, inverse of the inflation rate).

- Arbitrage and preferences drive the analysis of the equilibria of the benchmark model.
  1. Equilibria obey $\dfrac{\phi_{t+1}}{\phi_t} \leq r_{t+1} \leq \dfrac{1}{\beta}$ because preferences are quasilinear.
  2. Rule out a falling time path of consumption by arbitrage $\Longrightarrow$ rate of time preference $\geq$ gross real rate on government debt $\geq$ real return on cash.

Williamson (AER 2012): The FIs Market and Balance Sheet

- FIs are coalitions of Bs that form at the end of the CM subperiod.
  1. FIs receive deposits from Bs prior to the latter learning their type (*i.e.*, whether they will be monitored or not) and the DM opening.
  2. FIs buy cash and nominal bonds using the deposits of Bs.
  3. Assume free entry in banking $\Longrightarrow$ FIs earn zero profits.

- FIs form in the CM prior to the realization of ρ at date t.
  1. Subsequently, in the DM, unmonitored Bs withdraw m'/ρ of their deposits in the form of cash, where m is its real value $\Longrightarrow x_t = \beta\frac{\phi_{t+1}m'}{\phi_t\rho}$, where $m = \phi_t M_t$.
  2. At the same time, monitored Bs trade deposits equal to the change in their real cash and bond holdings $(m - m' + a - a')/(1-\rho)$, where a is the real value of nominal bonds $\Longrightarrow x_t = \beta\left[r_{t+1}\frac{a-a'}{1-\rho} + \frac{\phi_{t+1}(m-m')}{\phi_t(1-\rho)}\right]$, where $a = \phi_t B_t$.
  3. This trades leaves FIs with deposits equal to a' of the nominal bond, which are rebated to the monitored depositors in the CM of $t+1$.

- Once FIs pay off their depositors in the CM at $t+1$, they disband.

WILLIAMSON (AER 2012): THE FIS PROBLEM, PLENTIFUL LIQUIDITY

- Since Bs work in the CM, FIs maximize Bs' life time expected utility by choosing $\{m, a, m', a'\}$ s.t. $0 \leq m, a,\ 0 \leq m' \leq m$, and $0 \leq a' \leq a$,

$$\text{Max}_{\{m,\, a,\, m',\, a'\}} \Bigg[-\left(m + a\right) + \rho\,\mathcal{U}\left(\beta \frac{\phi_{t+1} m'}{\phi_t \rho}\right)$$

$$+ (1-\rho)\left[\mathcal{U}\left(\beta\left[r_{t+1}\frac{a-a'}{1-\rho} + \frac{\phi_{t+1}(m-m')}{\phi_t(1-\rho)}\right]\right) + \beta r_{t+1}\left(\frac{a'}{1-\rho}\right)\right]\Bigg].$$

- Since arbitrage demands $\dfrac{\phi_{t+1}}{\phi_t} \leq r_{t+1} \leq \dfrac{1}{\beta}$, solving the FI's problem yields
  1. **Plentiful Liquidity**: Monitored Bs demand for nominal bonds is always satisfied in the DM $\Longrightarrow$ efficiency sets their consumption $= x^*$; $a' = a - x^*$; $a \in \left[(1-\rho)x^*; \infty\right)$, and $r_{t+1} = \dfrac{1}{\beta}$, but cash is dominated in return by bonds, $\dfrac{\phi_{t+1}}{\phi_t} < r_{t+1} \Longrightarrow$ changes in cash's real return equates its supply and demand

$$\beta \frac{\phi_{t+1}}{\phi_t}\mathcal{U}'\left(\beta\frac{\phi_{t+1} m'}{\phi_t \rho}\right) = 1, \quad \text{where } m' = m.$$

  2. This Euler equation is an implicit money demand function.

WILLIAMSON (AER 2012): THE FIs PROBLEM, LIQUIDITY SHORTAGE

- Since Bs work in the CM, FIs maximize Bs' life time expected utility by choosing $\{m,\ a,\ m',\ a'\}$ s.t. $0 \le m,\ a,\ 0 \le m' \le m$, and $0 \le a' \le a$,

$$\text{Max}_{\{m,\ a,\ m',\ a'\}}\Bigg[-\big(m + a\big) + \rho\,\mathcal{U}\left(\beta\,\frac{\phi_{t+1}m'}{\phi_t\rho}\right)$$
$$+\ (1-\rho)\left[\mathcal{U}\left(\beta\left[r_{t+1}\frac{a-a'}{1-\rho} + \frac{\phi_{t+1}(m-m')}{\phi_t(1-\rho)}\right]\right) + \beta r_{t+1}\left(\frac{a'}{1-\rho}\right)\right]\Bigg].$$

- Since arbitrage demands $\dfrac{\phi_{t+1}}{\phi_t} \le r_{t+1} \le \dfrac{1}{\beta}$, solving the FI's problem yields a
  1. **Liquidity Shortage**: Cash and nominal bonds are dominated in rate of return by household time preference; also nominal bonds dominate cash in rate of return $\Longrightarrow$ shifts in the demand for nominal bonds moves inversely with r_{t+1}

$$\beta r_{t+1}\mathcal{U}'\left(\beta a\,\frac{r_{t+1}}{1-\rho}\right) = 1, \quad \text{where} \ \ m' = m, \ \ \text{and} \ \ a' = 0.$$

  2. Unmonitored Bs have access to a perfectly elastic supply of cash in the DM (*i.e.*, these Bs are on the flat part of their money demand schedules).
  3. A monitored B has a downward sloping nominal demand for bonds $\Longrightarrow$ entire supply of nominal bonds is traded in the CM and not carried over to $t+1$.

Williamson (AER 2012): The FIs Problem, Liquidity Trap

- Since Bs work in the CM, FIs maximize Bs' life time expected utility by choosing $\{m, a, m', a'\}$ s.t. $0 \le m, a, \ 0 \le m' \le m$, and $0 \le a' \le a$,

$$\text{Max}_{\{m,\, a,\, m',\, a'\}} \left[-\left(m + a\right) + \rho\,\mathcal{U}\left(\beta \frac{\phi_{t+1} m'}{\phi_t \rho}\right) \right.$$

$$\left. + \ (1-\rho)\left[\mathcal{U}\left(\beta\left[r_{t+1}\frac{a - a'}{1-\rho} + \frac{\phi_{t+1}(m - m')}{\phi_t(1-\rho)}\right]\right) + \beta r_{t+1}\left(\frac{a'}{1-\rho}\right)\right]\right].$$

- Since arbitrage demands $\dfrac{\phi_{t+1}}{\phi_t} \le r_{t+1} \le \dfrac{1}{\beta}$, solving the FI's problem yields a

  1. **Liquidity Trap**: Equate real returns on cash and nominal bonds, $\dfrac{\phi_{t+1}}{\phi_t} = r_{t+1}$, but the latter is dominated by $\beta^{-1} \Longrightarrow x_t$ is equated across monitored and unmonitored Bs in the DM, $\beta r_{t+1}\mathcal{U}'\left(\beta r_{t+1}\dfrac{m'}{\rho}\right) = 1$, where $\dfrac{m'}{\rho} = a + m$.
  2. Demand for cash and nominal demand is indeterminate, $m \ge \dfrac{\rho a}{1-\rho}$, but to hold sufficient cash to satisfy withdrawals by the unmonitored Bs $\Longrightarrow$ a "M&M theorem" for the asset side of FIs' balance sheets.
  3. FIs are observed holding "excess" cash reserves $\Longrightarrow a' = 0$.

WILLIAMSON (AER 2012): THE FIs PROBLEM, FRIEDMAN RULE

- Since Bs work in the CM, FIs maximize Bs' life time expected utility by choosing $\{m, a, m', a'\}$ s.t. $0 \leq m, a, \ 0 \leq m' \leq m$, and $0 \leq a' \leq a$,

$$\text{Max}_{\{m,\, a,\, m',\, a'\}} \left[-\left(m + a\right) + \rho \mathcal{U}\left(\beta \frac{\phi_{t+1} m'}{\phi_t \rho}\right) \right.$$
$$\left. + \ (1-\rho) \left[\mathcal{U}\left(\beta \left[r_{t+1} \frac{a - a'}{1-\rho} + \frac{\phi_{t+1}(m - m')}{\phi_t (1-\rho)} \right]\right) + \beta r_{t+1} \left(\frac{a'}{1-\rho}\right) \right] \right].$$

- Since arbitrage demands $\dfrac{\phi_{t+1}}{\phi_t} \leq r_{t+1} \leq \dfrac{1}{\beta}$, solving the FI's problem yields a
  1. **Friedman Rule**: Equate $\dfrac{\phi_{t+1}}{\phi_t} = r_{t+1} = \dfrac{1}{\beta} \Longrightarrow$ since FIs supplies of cash and nominal bonds are perfectly elastic, monitored and unmonitored Bs receive efficient consumption allocations in the DM $\Longrightarrow x^* = \dfrac{m'}{\rho} \leq \dfrac{m}{\rho}$ and $x^* = a + m - a'$ $(\leq a + m)$.
  2. The composition and size of FIs balance sheets are indeterminate $\Longrightarrow$ a "M&M theorem" for the asset side of FIs' balance sheets.

Williamson (AER 2012): The Government

- Benchmark is passive fiscal policy to focus on monetary policy experiment.
- Assume fiscal policy is always credible and time consistent.
  1. $\delta \in (-\infty, \infty) \Longrightarrow$ ratio of cash to cash plus nominal government bonds.
  2. $\mu \Longrightarrow$ fixed growth rate of cash plus nominal government bonds

     $$\Longrightarrow \mu = \frac{M_t + B_t}{M_{t-1} + B_{t-1}} = \frac{M_t}{\delta}\,\frac{\delta}{M_{t-1}} = \frac{M_t}{M_{t-1}}, \text{ where } \delta = \frac{M_t}{M_t + B_t}$$

     $\Longrightarrow \mu$ is the growth rate of cash given the share of cash in total government liabilities is fixed at δ, but a change in δ is a one off OMO.
- The government budget constraint becomes $\tau_0 = \phi_0 \dfrac{M_0}{\delta}$ at $t{=}0$ and for $t = 1, 2, \ldots, \infty$,

  $$\tau_t = \frac{\phi_t M_t}{\delta}\left[(1-\delta)\left(r_t - \frac{1}{\mu}\right) - \left(1 - \frac{1}{\mu}\right)\right],$$

  where the first term in brackets is the real interest cost of government debt and the second is seigniorage revenue per unit of cash.
- The fiscal authority adjusts lump sum taxes to “balance” the government’s budget $\Longrightarrow$ endogenous fiscal policy.

Williamson (AER 2012): Equilibrium Definition and Conditions

- **Definition 1**: Given a monetary policy (μ, δ), a *stationary* equilibrium with passive fiscal policy consists of real quantities of cash m, interest bearing assets a, a lump sum tax τ for $t = 1, 2, \ldots$, an initial tax τ_0, and a gross real interest rate r, such that
  1. m and a solve the FI's problem when $\frac{\phi_{t+1}}{\phi_t} = \frac{1}{\mu}$ and $r_{t+1} = r$,
  2. asset markets clear $\frac{m}{a} = \frac{\delta}{1-\delta}$, and
  3. the government budget constraints hold, where $\tau_0 = -\frac{m}{\delta}$ and
     $$\tau = \frac{m}{\delta}\left[(1-\delta)\left(r - \frac{1}{\mu}\right) - \left(1 - \frac{1}{\mu}\right)\right].$$
- A stationary equilibrium runs along the monetary growth path μ.
  1. Along this path inflation is always and everywhere a "monetary" phenomenon because $\frac{\phi_t}{\phi_{t+1}} = \mu$.
  2. Real side of the economy does not exhibit drift $\Longrightarrow r_{t+1} = r$.

Williamson (AER 2012): Equilibrium and Necessary Conditions

- Arbitrage: $\beta \leq \mu \Longrightarrow$ necessary condition that adds restrictions to the four equilibria.
- **Plentiful Liquidity**: Since $a \geq (1-\rho)x^*$; $\delta \leq m\Big/\Big[(1-\rho)x^* + m\Big] \Longrightarrow$ OMO is neutral because m is left unchanged, $\dfrac{\beta}{\mu}\mathcal{U}'\left(\dfrac{\beta m}{\mu\rho}\right) = 1$, given $\beta < \mu$; still raising μ lowers the δ at which the economy tips into an equilibrium with a liquidity shortage.
- **Liquidity Shortage**: OMOs are not neutral $\Longrightarrow$ add cash to the economy and ϕ falls proportionately, but $r \downarrow$ because the real value of the aggregate stock of government debt shrinks $\Longrightarrow$ FIs have less assets to back deposits unmonitored Bs use to trade in the DM $\Longrightarrow$ the real value of the last unit of $B \uparrow$ or B has greater liquidity premium.
- **Liquidity Trap**: Use $r\mu = 1$ (nominal rate = 0), the asset market clearing condition, and the relevant Euler equation to show $\dfrac{\beta}{\mu}\mathcal{U}'\left(\dfrac{\beta m}{\mu\delta}\right) = 1 \Longrightarrow m$ is not a function of r.
  1. Asset market clearing and $m \geq \rho a/(1-\rho)$ produces $\delta \geq \rho$, which is also a necessary condition of the liquidity trap equilibrium.
  2. The money demand function is not a downward sloping (*i.e.*, on its flat part) or monitored Bs are fully satiated with cash $\Longrightarrow$ the "too much cash" equilibrium.
- **Friedman Rule**: $\mu = \beta$ or $\beta r = 1$ for any $\delta \in (-\infty, \infty) \Longrightarrow$ multiplicity of Friedman rule equilibria, which explains its lack of use by CBs $\Longrightarrow$ are there frictions that would rule out this multiplicity?

Williamson (AER 2012): Summary

- Williamson (page 2601), "Fundamental to New Monetarist economics is the idea that the explicit roles played by particular assets in transactions, and how assets are intermediated, are critical to understanding the interaction among financial and monetary phenomena, quantities, and prices."
- There are four equilibria to study because of restrictions on β, r and $1/\mu$.
  1. Liquid assets earn a premium in the liquidity shortage equilibrium $\Longrightarrow$ this excess demand is the source of the monetary nonneutrality.
  2. The liquidity trap is an "extreme" liquidity shortage equilibrium $\Longrightarrow$ money demand is fully satiated signaling liquid assets that back FI deposits are being hoarded.
- Williamson argues the liquidity trap equilibrium of his model is better resembles the recent financial crisis than the Great Depression.
  1. The monetary policy response should be an OMO that removes cash from the economy instead of adding cash, which lowers the real rate.
  2. Sell interest bearing liquid assets into the economy to increase r to "redistribute" wealth to unmonitored Bs generating more real activity, which "jumps" the economy out of liquidity trap equilibrium.
- Assumptions about fiscal policy are needed to support the four equilibria.

Motivation: Williamson (JET, 2016)

- Take it as given that a CB is a FI, albeit one with monopoly power over fiat currency and its payment system(s).
    1. The monetary transmission mechanism, if it exists, starts with the CB altering the relative price of its liability, outside money, for a private short term security, inside money.
    2. The CB is able to alter this relative price only when short rates are strictly positive.

- What monetary policy tools are available to CB when short rates are at or near the zero lower bound (ZLB)?

- Several CBs have engaged in quantitative easing (QE) using large scale purchase programs (LSAPs) $\Longrightarrow$ at the ZLB, a CB operates on the quantity margin instead of the interest rate margin.
    1. LSAPs: generic name for the monetary policy tool that has a CB buying lots of financial securities.
    2. QE: a name given to specific LSAP policy action by a CB.
    3. For example, the Federal Open Market Committee (FOMC) has run two QEs policies in which it has bought US Treasury securities and MBS and a third QE in which only the former were purchased.

WILLIAMSON (JET, 2016): LSAP/QE MONETARY TRANSMISSION

- If a QE policy offers long dated securities in exchange
  1. for, say, shorter term sovereign debt, a CB is transforming the maturity structure of the balance sheets of private financial firms.
  2. This is an activity most often associated with private FIs.

- Are QE policies just a CB taking on the role of private FIs, or is there a LSAP-monetary transmission mechanism?

- LSAP-monetary transmission seems feasible within preferred habit models.
  1. Asset markets are segmented because different investors “prefer” or demand different securities $\Longrightarrow$ passive investors prefer some securities over others, say longer dated ones more than short term.
  2. Market segmentation is a friction that rules out asset substitution $\Longrightarrow$ rates adjust to clear markets in which two types of investors trade.
  3. Segmented asset markets are managed by arbitragers $\Longrightarrow$ maximize wealth by demanding risk premia to absorb securities in excess supply.
  4. The supply of securities is an exogenous process.

- Preferred habit models predict a CB can alter risk premium by buying and selling securities; see Vayanos and Villa (2009).

WILLIAMSON (JET, 2016): A NEW MONETARIST APPROACH TO LSAPS

- The model is an extension of Williamson (AER, 2012).
- Williamson adapts a KM-type collateral constraint to model the pledgeability of government debt with different maturities.
  1. There are short and long maturity government securities.
  2. Short maturity debt has lower costs in backing FI deposits $\Longrightarrow$ FIs suffer larger haircuts selling long dated government debt to other FIs to pay off deposits.
  3. Assume θ_S (θ_L) is the fraction of a unit of short (long) maturity debt a monitored B can consume.
  4. $\Longrightarrow$ An upward sloping yield curve needs $\theta_S < \theta_L$.
  5. $\Longrightarrow$ Long dated government debt is less pledgeable or suffers larger haircuts as collateral than short dated government debt.
- A liquidity shortage depends on the value and composition of government liabilities, which fiscal policy sets, not the CB, in Williamson's model.
- Bs and FIs have differential demands for short and long dated government debt $\Longrightarrow$ OMOs that alter the composition of these securities can produce shifts in nominal and real rates and inflation.

Williamson (JET, 2016): Fiscal and Monetary Policy

- Williamson (p. 10), "The specification of the relationship between fiscal and monetary policy will be critical to how this model works."
- The central bank is not independent of the fiscal authority.
  1. The evidence is not just that the model is analyzed by consolidating the budget constraints of the CB and fiscal authority.
  2. The CB rebates income, which could be negative, from its portfolio of government securities to the fiscal authority.
  3. $\Longrightarrow$ The fiscal authority backs the CB with taxes levied on Bs.
- The model equates monetary policy and the monetary operating system.
  1. The floor system sets the policy rate equal to interest on reserves (IOR).
  2. When the policy rate dominates IOR, a CB is running a channel system (and the policy rate is less than the discount rate).
  3. A CB's choice of operating system has implications for financial market efficiency, monetary policy control of the price level (or inflation), and fiscal–monetary policy interactions.

Williamson (JET, 2016): The Channel System

- Under IOR, banks earn interest income on the cash reserves they hold.
  1. IOR is equivalent to a CB issuing debt, but that debt is not necessarily a (close) substitute to short dated government debt.
  2. Treasury & CB debt are (not) substitutes under a floor (channel) system.
- A channel system sets IOR $<$ policy rate $\Longrightarrow$ return on short term government debt.
  1. An opportunity cost to hold reserves $\Longrightarrow$ FIs economize on reserves.
  2. FIs would rather hold securities that at least earn the policy rate, which is a wedge between credit and money markets.
  3. The inefficiency is a monetary transmission mechanism in Williamson's model $\Longrightarrow$ OMOs alter the composition of FI balance sheets.
- Policy Experiment: CB trades short government debt for cash.
  1. The OMO lowers the real return on cash $\Longrightarrow$ inflation and the real return on short government debt rise
  2. $\Longrightarrow$ incentives for FIs to hold more short government debt.
- Policy Experiment: CB trades short government debt for long government debt.
  1. This is QE in Williamson's model $\Longrightarrow$ the binding constraint is that there is a shortage of liquidity or short government debt.
  2. QE alters the share of "good" collateral on FI balance sheets $\Longrightarrow$ generates an increase in the real return on short government debt and cash, which is an incentive for monitored Bs to hold more cash.
  3. The ZLB is a symptom or an endogenous response to an "extreme" liquidity shortage rather than being a constraint on CB policymakers.

Williamson (JET, 2016): The Floor System

- The policy rate equals IOR under the floor system.
- In Williamson's model, the price or return on money and short government debt are the same $\Longrightarrow$ OMOs can swap reserves for long government debt in a floor system because short government debt and CB debt are substitutes.
- Under a floor system, a CB has an additional policy tool.
  1. The CB can issue sufficient debt to purchase the entire stock of long government debt.
  2. This is optimal policy $\Longrightarrow$ transform the maturity structure of FIs in Williamson's model by giving them short term interest bearing assets, which eliminates the liquidity shortage in financial markets.
  3. This result relies on all FIs accepting CB debt as a medium with which to clear their accounts.
- Under a floor system, Williamson's model predicts a QE program has similar effects as found for QE and the channel system at the ZLB $\Longrightarrow$ short debt and cash have equal returns.
- The results of QE are conditional on way in which IOR payments to FIs are financed whether the CB runs a floor or a channel system $\Longrightarrow$ fiscal policy matters for the analysis of LSAPs.

Williamson (JET, 2016): Floor & Channel Systems and IOR

- Friedman, M. 1960, A Program for Monetary Stability, New York: Fordham University Press.
- Goodfriend, M. 2002, "Interest on Reserves and Monetary Policy," Federal Reserve Bank of New York Economic Policy Review, 8(1), 13–29.
- Kashyap, A.K., J.C. Stein. 2012, "The Optimal Conduct of Monetary Policy with Interest on Reserves," *American Economic Journal: Macroeconomics* 4, 266–282.
- Keister, T., A. Martin, J. McAndrews. 2008, "Divorcing Money from Monetary Policy," Federal Reserve Bank of New York Economic Policy Review, 14(2), 41–56.
- Stein, J.C. 2012, "Monetary Policy as Financial-Stability Regulation," *Quarterly Journal of Economics* 127, 57–95.

Williamson (JET, 2016): Summary

- Williamson develops a model with a financial friction.
  1. Private debt only circulates with positive finite value if it is backed by short government debt.
  2. This economy exhibits monetary nonneutrality if there is a shortage of short government debt $\Longrightarrow$ there is an excess demand for liquidity.

- When the government issues debt with long and short maturities, there is an inefficiency in Williamson's model.
  1. Long government debt demands a higher return because it is less liquid $\Longrightarrow$ suffers larger haircuts by assumption.
  2. In a liquidity shortage, optimal policy is for the CB to buy the entire stock of long term government debt, which shortens and liquefies FI balance sheets.

- Williamson's model assumes long term government debt is inefficient $\Longrightarrow$ whether it exists to solve a different problem facing fiscal policy is beyond the scope of this analysis.

Introduction: Review

- The BGG and KM classes of DSGE models focus on the banking accelerator.

- *Financial Accelerator*: An expansionary monetary policy shock increases the MP_K, which raises the value of capital loosening financing constraints facing entrepreneurs $\Longrightarrow$ reduces the external finance premium.

- This section reviews papers that focus on the second part of the financial accelerator to study the role of government policy during a financial crisis.
    1. Increased "bank capital" loosens financing constraints facing FIs $\Longrightarrow$ credit supply can increase.
    2. The external finance premium is an incentive for FIs to issue more loans.
    3. The FI holds more assets on its balance sheet on a fixed base of net worth $\Longrightarrow$ increase leverage.
    4. The more leveraged are FIs the larger is the drop in the value of the asset side of their balance sheet in response to an aggregate shock that drops the price of (physical) capital
    5. FI credit creation shuts down $\Longrightarrow$ the government can stabilize the economy by issuing the credit the FIs cannot.

- This assumes away FIs engage in strategic behavior when making decisions about leverage.

Introduction: Papers Covered in this Section

- Gertler and Karadi (JME, 2011) construct a NKDSGE model that include finite-lived FIs in an otherwise standard NKDSGE model.
  1. A FI transforms deposits (*i.e.*, its short term liabilities) from households into its long term assets by lending to firms needing capital to produce output.
  2. $\Longrightarrow$ Incentive is for FIs to issue more loans when these assets offer excess returns compared with the riskless return on deposits.
  3. FIs exploit the excess returns on loans by creating more of these assets per unit of liability (*i.e.*, deposits and/or "bank capital"), which increases profit per loan by lowering funding costs.
  4. When a contractionary shock shuts down the FI sector's ability to engage in maturity transformation, the CB fills in the missing market by lending to firms $\Longrightarrow$ assumption is the CB has "deep pockets."
  5. The CB only lends to firms in states in which FIs cannot because it incurs greater costs to accomplish this task than FIs by assumption.

- Gertler, Kiyotaki, and Queralto (JME, 2012) work with a DSGE similar to Gertler and Karadi (GK) NKDSGE but sans NK nominal frictions.
  1. Gertler, Kiyotaki, and Queralto (GKQ) separate FI net worth from the equity issued by these firms $\Longrightarrow$ the price of FI equity and physical capital also differ.
  2. A FI funded by deposits, net worth, and equity has an additional margin on which to create leverage and tap excess returns to generate profits.
  3. A financial crisis is modeled as an infrequent and exogenous disaster shock $\Longrightarrow$ "expectations" of this shock create beliefs about the probability of future financial crisis.

Introduction: Papers Covered in this Section, cont.

- Leverage is neither the cause nor the source of financial crises in the GK and GKQ models $\Longrightarrow$ uninsurable aggregate shocks are, which motivates a CB to engage in credit policy to replace the missing inside money.

- Farhi and Tirole (AER-PnP, 2009) argue that assuming leverage in and of itself is benign is dangerous $\Longrightarrow$ leverage creates incentives for FIs to act strategically.

- The incentives are associated with FI expectations of the policy responses of government agencies to a financial crisis.

- Farhi and Tirole (FT) label CB interest rate policy a non-targeted public policy.
  1. Example: A CB changing its policy rate may affect inflation (and/or expectations about it) and stabilize the business cycle, but
  2. also can alter the value of assets and liabilities on FI balance sheets.
  3. $\Longrightarrow$ Creates incentives for FIs to increase the interest rate sensitivity of their balance sheets.
  4. FIs act strategically to increase the chances the CB will lower rates during a financial crisis "bailing out" leveraged FIs.
  5. Monetary policy becomes time-inconsistent $\Longrightarrow$ once a financial crisis begins, a CB has an incentive to lower its policy rate to "save" FIs.
  6. Macro-prudential policies yield better ex ante and ex post outcomes w/r/t leveraged FIs and uninsurable shocks, according to FT.
  7. The complete theory/model is presented in Farhi and Tirole (2013, "Collective Moral Hazard, Maturity Mismatch, and Systemic Bailouts," *American Economic Review* 102, 60–93) .

Gertler and Karadi (JME 2011): Introduction

- Goal: construct a NKDSGE model in which the CB operates unconventional policy.

- The NKDSGE model is standard except for FIs.
  1. The household consists of a unit mass of members, where a fraction $1-f$ are workers and the remaining are FIs.
  2. FIs are finite-lived with a *iid* probability $\theta \in (0,\ 1)$ of continuing from date t to $t+1 \Longrightarrow$ a former FI becomes a worker and new FIs were workers previously.
  3. FIs take one-period deposits (debt) from households and lend to firms that lack the funds to buy the capital needed for production.
  4. This is an example of maturity transformation $\Longrightarrow$ the FI creates long term assets for its balance sheet out of its short term liabilities.
  5. When FI net worth falls, loans to goods producing firms drop $\Longrightarrow$ the credit market contracts, which produces a drop in real activity (*i.e.*, a recession).
  6. Firms receive loans from the CB only in this state of the world $\Longrightarrow$ FIs are more efficient at loan creation than the CB in all states of the world.
  7. CB credit policy mitigates recessions by boosting loans to firms to stabilize the business cycle $\Longrightarrow$ a policy independent of the level of the policy rate in which the CB shares risk with savers and borrowers when FIs cannot.

- The CB stabilizes output by extending loans to firms.
  1. The CB intermediates between savers and borrowers when FIs cannot.
  2. Assume the CB can expand its balance sheet without limit, but GK impose a social cost on the CB when it engages in credit allocation.
  3. Credit policy predicated on the central bank issuing debt, which suggests CB credit policy is equivalent to fiscal policy.

GK's NKDSGE Model: The Household

- The household maximizes its expected discounted lifetime utility function

$$\mathbf{E}_t\left\{\sum_{j=0}^{\infty}\beta^j\Big[\ln(c_{t+j}-hc_{t+j-1}) - \frac{\chi}{1+\varphi}n_{t+j}^{1+\varphi}\Big]\right\},\quad \beta,h\in(0,1),\quad \chi,\varphi>0,$$

subject to the budget constraint

$$c_t + d_{t+1} = w_t n_t + (1+r_{d,t})d_t + \mu_t + \tau_t,$$

and the non-negativity constraints $c_t,\ n_t,\ d_{t+1}\geq 0$, where $\mathbf{E}_t\{\cdot\}$ is the mathematical expectations operator, c_t is consumption, n_t is labor supply, b_{t+1} is the stock of one-period real deposits the household leaves with a FI from date t into $t+1$, w_t is the real wage, r_t is the real riskless return on deposits, τ_t is a lump sum tax, and μ_t is the endowment the household gives to its members starting a FI at date t.

GK's NKDSGE Model: The Household's Optimality conditions

- The FONCs w/r/t c_t, n_t, and b_{t+1} yield the forward-looking marginal utility of consumption for internal consumption habit preferences

$$\lambda_t = \frac{1}{c_t - hc_{t-1}} - \beta h \mathbf{E}_t \left\{ \frac{1}{c_{t+1} - hc_t} \right\},$$

the real wage measured in utils equals the marginal disutility of work

$$\lambda_t w_t = \chi n_t^{\varphi},$$

and the cost of giving up a unit of c_t equals the discounted riskless return to holding an extra unit of d_{t+1} measured in expected utils

$$\lambda_t = \beta \left(1 + r_{d,t+1}\right) \mathbf{E}_t \lambda_{t+1},$$

where the Lagrange multiplier on the household budget constraint is denoted λ_t.

GK's NKDSGE Model: Capital Producing Firms

- Intermediate goods firms buy (or rent) capital, k_t, from firms that produce the durable good.
- Capital goods firms generate a flow of new capital, $x_{N,t}$, and "fix" k_t worn out by producing intermediate goods, $\delta(u_t)\xi_t k_t$, to generate gross investment, x_t.
  1. $\Longrightarrow x_t = x_{N,t} + \delta(u_t)\xi_t k_t$, where depreciation, $\delta(\cdot)$, is an increasing function of intermediate goods firms' utilization, u_t, of efficiency units of capital, $\xi_t k_t$.
  2. Denote the price of new capital with q_t and normalize the price of renovated capital to one $\Longrightarrow$ equate its value to that of the numeraire good, a unit of c_t.
- Subject to the law of motion of investment, a capital goods firms maximizes its discounted expected profits w/r/t $x_{N,t}$

$$\text{Max}_{x_{N,t}}\, \mathbf{E}_t \left\{ \sum_{j=0}^{\infty} \beta^j \Lambda_{t+j,t} \left[(q_{t+j} - 1)x_{N,t+j} - \mathcal{X}\left(\frac{x_{N,t+j} + x_{SS}}{x_{N,t+j-1} + x_{SS}} \right) \left[x_{N,t+j} + x_{SS} \right] \right] \right\},$$

  1. where $\Lambda_{t+j,t} = \dfrac{\lambda_{t+j}}{\lambda_t}$ because the household owns these firms, x_{SS} is steady state investment, and adjustment costs of changing new investment, $\mathcal{X}(\cdot)$, restricted by $\mathcal{X}(1) = \mathcal{X}'(1) = 0$, and $\mathcal{X}''(1) > 0$.
  2. The adjustment cost function $\mathcal{X}(\cdot)$ is responsible for Tobin's q, $q_t > 1$.
  3. $\Longrightarrow$ Placing adjustment costs on $x_{N,t}$ lets capital goods firms generate positive profits that are returned to households as dividends ($\Longrightarrow$ implicit is capital goods firms repair all k_t depreciated during production of the intermediate good at cost or for zero profit).

GK's NKDSGE Model: Capital Producing Firms' Optimality Condition

- The FONC with respect to $x_{N,t}$ yields the Tobin's q-capital pricing equation

$$q_t - 1 = \mathcal{X}\left(\frac{x_{N,t} + x_{SS}}{x_{N,t-1} + x_{SS}}\right) + \left[\frac{x_{N,t} + x_{SS}}{x_{N,t-1} + x_{SS}}\right]\mathcal{X}'\left(\frac{x_{N,t} + x_{SS}}{x_{N,t-1} + x_{SS}}\right)$$
$$- \beta \mathbf{E}_t\left\{\Lambda_{t+1,t}\left[\frac{x_{N,t+1} + x_{SS}}{x_{N,t} + x_{SS}}\right]^2\mathcal{X}'\left(\frac{x_{N,t+1} + x_{SS}}{x_{N,t} + x_{SS}}\right)\right\}.$$

- The price of capital exceeds the price of a unit of c_t because
  1. of the costs associated with adjusting new investment, $\mathcal{X}(\cdot)$, plus
  2. the unit marginal cost, $\mathcal{X}'(\cdot)$, generated by adding a unit of $x_{N,t}$, net
  3. of the discounted expected adjustment costs foregone at date $t+1$, given an additional unit of new investment was installed at date t.

GK's NKDSGE Model: Intermediate Goods Firms, I

- Intermediate goods firms face competitive loan, labor, capital, and goods markets.

- FIs receive "equity" from intermediate goods firms in return for loans at the end of date $t-1$ that the latter use to buy k_t.
    1. A unit of equity, s_t, issued at the end of date $t-1$ is a claim on a unit of k_t, where the price per unit of s_t = price per unit of k_t.
    2. $\Longrightarrow q_t s_t = q_t k_t$, otherwise a intermediate goods firm would have issued too little or too much s_t to obtain k_t.
    3. GK assume equity always pays (*i.e.*, no financial frictions inhibit intermediate goods firms) $\Longrightarrow$ equity is a state contingent security conditional on loan supply.

- Given k_t, a TFP shock z_t, and a shock to capital efficiency ξ_t, the intermediate good, y_t is produced using a CRS-labor neutral technology mixing labor, ℓ_t, with efficiency units of capital, $\xi_t k_t$, operating at capacity u_t, $z_t[u_t \xi_t k_t]^{\alpha} \ell_t^{1-\alpha} \Longrightarrow$ intermediate goods firms aim to $\text{Max}_{\{u_t,\ \ell_t\}} p_{M,t} y_t + q_t \xi_t k_t - \delta(u_t)\xi_t k_t - w_t \ell_t$, where
    1. $p_{M,t}$ = the relative price of y_t to $c_t \Longrightarrow p_{M,t} \neq 1$ because of $\mathcal{X}(\cdot)$.
    2. FONC w/r/t u_t: $p_{M,t}\alpha y_t / u_t = \delta'(u_t)\xi_t k_t \Longrightarrow$ the marginal revenue product of working k_t more intensively = increased depreciation of k_t in efficiency units.
    3. FONC w/r/t ℓ_t: $p_{M,t}(1-\alpha) y_t / \ell_t = w_t \Longrightarrow$ the marginal revenue product of ℓ_t = cost of hiring additional ℓ_t, which is the real wage.

GK's NKDSGE Model: Intermediate Goods Firms, II

- Conditional on the supply of loanable funds, intermediate goods firms earn zero profits in all states of the world.
- Since FIs face constraints on their balance sheets and intermediate goods firms do not, they have perfectly elastic demand for loans.
- Intermediate goods firms repay date t loans with "dividends" generated by date $t+1$ production, $p_{M,t+1}\alpha y_{t+1}/k_{t+1} + q_{t+1}\xi_{t+1} - \delta(u_{t+1})\xi_{t+1}$.
- Since these date t loans cost q_t per unit of k_t, the date $t+1$ return to capital is

$$1 + r_{k,t+1} = \left[\frac{p_{M,t+1}\alpha y_{t+1}/(\xi_{t+1}k_{t+1}) + q_{t+1} - \delta(u_{t+1})}{q_t}\right]\xi_{t+1}.$$

- The ex post return to capital is scaled by the shock to the efficiency units of capital.
  1. A positive ξ_{t+1} shock raises the return to capital one for one, but lowers the marginal revenue (of efficiency units) of capital.
  2. $\Longrightarrow$ The elasticity of the return to capital w/r/t ξ_{t+1} = one minus the ratio of the marginal revenue product (of efficiency units) of capital to $1 + r_{k,t+1}$.

GK's NKDSGE Model: Final Goods Sector

- The final goods sector is where NK nominal price rigidities reside.
- Final goods firm buy intermediate goods in a competitive market, but
  1. these firms sell relabeled intermediate goods into a monopolistically competitive market final goods market.
  2. Final good price dynamics are governed by Calvo staggered pricing in which there is full indexation to inflation.

GK's NKDSGE Model: Financial Intermediaries

- FIs borrow from households, accumulate wealth for their shareholders, and use these liabilities and assets to issue loans to intermediate goods firms.

- Wealth is accumulated by FIs when $\mathbf{E}_t r_{k,t+1} > r_{d,t+1} \Longrightarrow$ the expected return on loans is greater than the riskless return on deposits.
    1. At any date t, the mth FI's balance sheets consists of liabilities, $d_{m,t}$ plus wealth, $n_{m,t-1}$, and assets, $q_{t-1}s_{m,t-1}$, carried from date $t-1$ into date t $\Longrightarrow d_{m,t} + n_{m,t-1} = q_{t-1}s_{m,t-1}$.
    2. FI wealth can be thought of as either net worth or equity issued to the household, which is often identified with "bank capital."
    3. The flow into $n_{m,t}$ consists of the return to loans, $r_{k,t}q_{t-1}s_{m,t-1}$, net of the cost of deposits, $(1+r_{d,t})d_{m,t} \Longrightarrow$ combine with the balance sheet constraint to obtain the law of motion $n_{m,t} = (r_{k,t} - r_{d,t})q_{t-1}s_{m,t-1} + (1+r_{d,t})n_{m,t-1}$.
    4. Two factors generate growth in FI wealth $\Longrightarrow$ the riskless return on existing wealth, $(1+r_{d,t})n_{m,t-1}$, and the risk premium on loans, $r_{k,t} - r_{d,t}$.
    5. Government liquidity injections do not appear on the asset side of FI balance sheets because the NKDSGE model is missing a monetary aggregate.

- Under complete Arrow-Debreu markets, arbitrage forces equality on the expected risky and riskless returns $\Longrightarrow \mathbf{E}_t\{r_{k,t+j} - r_{d,t+j}\} = 0$.

- If financial markets are incomplete, there are unexploited arbitrage opportunities, $\mathbf{E}_t\{r_{k,t+j} - r_{d,t+j}\} > 0$, tied to supply constraints on FI loan creation.

GK's NKDSGE Model: FI's Dynamic Program

- $\mathcal{V}_{m,t}$ = mth FI's lifetime expected discounted wealth $\Longrightarrow$ current value of FI's program.
- Given θ is the probability a FI continues operating from one period to the next and FIs are owned by the household,

$$\mathcal{V}_{m,t} \;=\; \text{Max}\, \beta(1-\theta)\mathbf{E}_t \left\{ \sum_{j=0}^{\infty} \left(\beta\theta\right)^j \Lambda_{t+1+j,t} n_{m,t+1+j} \right\},$$

where $n_{m,t+1} = (r_{k,t+1} - r_{d,t+1})q_t s_{m,t} + (1 + r_{d,t+1})n_{m,t}$.

GK's NKDSGE Model: FI Optimal Behavior

- GK conjecture that $\mathcal{V}_{m,t}$ is a linear function of the current market value of loans and FI wealth $\Longrightarrow \mathcal{V}_{m,t} = \nu_{m,t} q_t s_{m,t} + \eta_{m,t} n_{m,t}$.

1. $\nu_{m,t} = \beta(1-\theta)\mathbf{E}_t\left\{\sum_{j=0}^{\infty}\left(\beta\theta\right)^j \Lambda_{t+1+j,t}\left(r_{k,t+1+j} - r_{d,t+1+j}\right)\frac{q_{t+1+j}s_{m,t+1+j}}{q_t s_{m,t}}\right\}$

   $= \beta(1-\theta)\mathbf{E}_t\left\{\Lambda_{t+1,t}\left(r_{k,t+1} - r_{d,t+1}\right) + \beta\theta\frac{q_{t+1}s_{m,t+1}}{q_t s_{m,t}}\nu_{m,t+1}\right\} \Longrightarrow$ the mth FI's discounted expected benefit of an extra unit of $q_t s_{m,t}$, all else constant.

2. $\eta_{m,t} = \beta(1-\theta)\mathbf{E}_t\left\{\sum_{j=0}^{\infty}\left(\beta\theta\right)^j \Lambda_{t+1+j,t}\left(1 + r_{d,t+1+j}\right)\frac{n_{m,t+1+j}}{n_{m,t}}\right\}$

   $= \beta(1-\theta)\mathbf{E}_t\left\{\Lambda_{t+1,t}\left(1 + r_{d,t+1}\right) + \beta\theta\frac{n_{m,t+1}}{n_{m,t}}\eta_{m,t+1}\right\} \Longrightarrow$ the mth FI's discounted expected benefit of adding a unit of $n_{m,t}$, all else constant.

3. With complete state contingent claims markets, the arbitrage condition $\beta\mathbf{E}_t\left\{\Lambda_{t+1+j,t}\left(r_{k,t+1} - r_{d,t+1}\right)\right\} = 0$ tells the mth FI to stop issuing loans.

GK's NKDSGE Model: Constraints on FI Optimal Behavior, I

- FIs want to issue more loans when $\beta \mathbf{E}_t \left\{ \Lambda_{t+1,t} \left(r_{k,t+1} - r_{d,t+1} \right) \right\} > 0$.

- GK assume the mth FI can abscond with $\lambda \in (0, 1)$ of the market value of its loans $\Longrightarrow$ the FI is closed by depositors who obtain the remaining $(1 - \lambda) q_t s_{m,t}$ of FI assets.
  1. Depositors only lend to FIs with lifetime expected discounted wealth greater than or equal to the fraction of the market value of its loans with which the FI can flee town $\Longrightarrow$ The incentive compatibility constraint depositors impose on the mth FI is $\mathcal{V}_{m,t} = \nu_{m,t} q_t s_{m,t} + \eta_{m,t} n_{m,t} \geq \lambda q_t s_{m,t}$.
  2. When the mth FI is wealth constrained, $\nu_{m,t} q_t s_{m,t} + \eta_{m,t} n_{m,t} = \lambda q_t s_{m,t} \Longrightarrow$

$$q_t s_{m,t} = \frac{\eta_{m,t}}{\lambda - \nu_{m,t}} n_{m,t}.$$

  3. A necessary condition for a wealth constrained FI to issue new loans is $\nu_{m,t} \in (0, \lambda) \Longrightarrow$ when $n_{m,t} > 0$, as $\nu_{m,t} \longrightarrow \lambda$ continuing as a FI is more valuable than bolting with $\lambda q_t s_{m,t}$, which holds trivially as $\nu_{m,t} > \lambda$.
  4. Increasing loans, $s_{m,t}$, on a fixed base of wealth, $n_{m,t}$, induces the mth FI to run on its depositors $\Longrightarrow$ either $\eta_{m,t}$ rises, $\nu_{m,t}$ falls, or a combination of both.
  5. Define $\phi_{m,t} \equiv \dfrac{\eta_{m,t}}{\lambda - \nu_{m,t}}$, which GK call the mth FI's leverage ratio $\Longrightarrow$ benefits to the FI of taking off with $\lambda q_t s_{m,t}$ equals the discounted expected loss of foregone FI wealth, or $q_t s_{m,t} = \phi_{m,t} n_{m,t}$.

GK's NKDSGE Model: Constraints on FI Optimal Behavior, II

- Use the leverage ratio to revise the mth FIs law of motion of wealth,

$$n_{m,t+1} = \Big[(r_{k,t+1} - r_{d,t+1})\phi_t + (1 + r_{d,t+1})\Big]n_{m,t}.$$

- $\Longrightarrow$ A one percent increase in $n_{m,t+1}$ is possible with a smaller risk premium given larger FI leverage (ex post).

- The leverage ratio also yields recursions
  1. for the growth rate of leveraged wealth, $\dfrac{q_{t+1+j}s_{m,t+1+j}}{q_t s_{m,t}} = \dfrac{\phi_{t+1+j}n_{m,t+1+j}}{\phi_t n_{m,t}}$,
  2. and the growth rate wealth $\dfrac{n_{m,t+1+j}}{n_{m,t}} = (r_{k,t+1} - r_{d,t+1})\phi_t + (1 + r_{d,t+1})$.
  3. GK claim that these recursions show ϕ_t has no dependence on the attributes of the mth FI $\Longrightarrow q_t s_t = \phi_t n_t$ is the result of aggregating across all FIs.

GK's NKDSGE Model: Constraints on FI Optimal Behavior, III

- Aggregate FI wealth can be decomposed into the wealth of surviving FIs, $n_{S,t}$, and new FIs, $n_{N,t} \Longrightarrow n_t = n_{S,t} + n_{N,t}$, or

$$n_t = \theta\Big[(r_{k,t} - r_{d,t})\phi_{t-1} + (1 + r_{d,t})\Big]n_{t-1} + n_{N,t}.$$

1. Changes in $r_{k,t} - r_{d,t}$ and/or ϕ_{t-1} fall on wealth of existing FIs, $n_{S,t}$.
2. FIs departing the market at date t return with $(1-\theta)q_t s_{t-1}$ resources to the household (remember the probability of a FI exit is *IID*).
3. GK assume the household gives $\dfrac{\omega}{1-\theta}$ of these resources to start-up FIs, where $\omega \in (0, 1) \Longrightarrow n_{N,t} = \omega q_t s_{t-1}$.

- Substitute for $n_{N,t}$ using $\omega q_t s_{t-1}$ to obtain the law of motion for aggregate FI wealth

$$n_t = \theta\Big[(r_{k,t} - r_{d,t})\phi_{t-1} + (1 + r_{d,t})\Big]n_{t-1} + \omega q_t s_{t-1}.$$

1. Steady state leverage ratio is $\phi^* = \dfrac{q^* s^*}{n^*} = \dfrac{1 - \theta(1 + r_d^*)}{\omega + \theta(r_k^* + r_d^*)} \Longrightarrow$ a small ω is consistent with $\phi^* > 1 \Longrightarrow$ few household resources are needed to start up a FI.
2. If $(1 + r_d^*)^{-1} > \theta,\ \phi^* > 0 \Longrightarrow$ positive steady state leverage needs the market discount on FI wealth $>$ survival probability of FI.

GK's NKDSGE Model: Central Banks and Credit Policy, I

- Consider a CB willing to lend to firms by borrowing from households.
  1. The value of assets the CB intermediates is denoted $q_t s_{CB,t}$.
  2. $\Longrightarrow$ In the aggregate, intermediated assets are denoted $q_t s_t = q_t s_{FI,t} + q_t s_{CB,t}$, where $q_t s_{FI,t}$ is the value of assets intermediate by FIs.
- The CB can issue debt to households at its riskless, $r_{CB,t}$, to lend to firms at rate $r_{k,t}$ $\Longrightarrow$ by arbitrage $r_{CB,t} = r_{d,t}$.
  1. GK impose a dead weight loss of τ per unit of loan on the economy when the CB acts as an FI $\Longrightarrow$ FIs are more efficient at intermediating than is the CB.
  2. GK also assume the CB always repays households $\Longrightarrow$ no adverse selection or moral hazard problems.
  3. The lack of financial frictions facing the CB means it is not "balance sheet constrained" $\Longrightarrow$ the CB has deep pockets.
- Next, GK propose an alternative unconventional monetary policy scheme $\Longrightarrow$ the CB borrows from FIs instead.
  1. The CB gives FIs its *IOU* or debt, $b_{G,t}$ that pays $r_{CB,t}$ $(= r_{d,t})$.
  2. FIs have "excess" reserves, which can be lent to firms at $r_{k,t}$.
  3. Reserves can be expanded by the CB without limit, according to GK $\Longrightarrow$ the CB is not constrained by its balance sheet.

GK's NKDSGE Model: Central Banks and Credit Policy, II

- GK define the fraction of assets intermediate by the CB to be $\psi_t \in (0, 1)$ $\Longrightarrow q_t s_t = q_t s_{FI,t} + \psi_t q_t s_t$.
  1. $\Longrightarrow q_t s_{CB,t} = \psi_t q_t s_t$, where $(r_{k,t} - r_{d,t})b_{G,t-1}$ is the net income the CB passes to the Treasury for acting as an intermediary.
  2. This revenue must be recorded on the (consolidated) government budget constraint.

- Since the CB is intermediating, it can engage in leverage, $\phi_{CB,t}$, as do FIs.
  1. Remember $q_t s_{FI,t} = \phi_t n_t \Longrightarrow \phi_{CB,t} = \dfrac{1}{1 - \psi_t}\phi_t$.
  2. As ψ_t increases, $\phi_{CB,t}$ also rises $\Longrightarrow$ CB leverage moves with the share of the market for intermediation that it controls.

GK: SUMMARY, I

- GK aim to judge the impact of CB emergency loan programs (or unconventional monetary policy actions) to limit the effects of a financial crisis.
- A canonical NKDSGE model is wrapped around a version of the Gertler and Kiyotaki (AER, 2015) model of financial crises.
  1. Firms are monopolistic competitors $\Longrightarrow$ Calvo staggered price setters.
  2. FIs face "liquidity preference shocks" $\Longrightarrow$ a FI use its new worth to decide whether to continue or consume a fraction of its current assets (*i.e.*, loans).
- The CB uses a Taylor rule to operate monetary policy, but can also issue debt to FIs.
  1. $\Longrightarrow$ Re-liquefying FI balance sheets loosens the ICCs that bind more tightly in response to an exogenous "financial market shock" to FI asset quality.
  2. The social cost is a loss of real resources because the CB is inefficient at intermediating compared with FIs.
- GK calibrate their NKDSGE to previous studies and to sample data.
  1. Steady state interest rate spread tied to risk spreads $\Longrightarrow$ GK model liquidity crises not insolvency of the FI sector.
  2. Fraction of assets FIs grab when "taking the money and running" is about 38% $\Longrightarrow$ aggregate FI leverage ratio $\phi^* \approx 4.0$.
  3. $\Longrightarrow$ FI balance sheets are sensitive to a shock that drives FI net worth lower.

GK: Summary, II

- GK report household welfare is higher after the CB issues debt to FIs in response to a negative financial market shock.
    1. This result is sensitive to the dead weight loss imposed on the economy by the CB engaging in financial intermediation.
    2. However, welfare gains occur whether or not the CB's policy rate is near zero.

- Several issues are left for future research by GK.
    1. Steady state leverage is exogenous for FIs $\Longrightarrow$ the FI sector is not at risk only a fraction of FIs.
    2. Financial market shocks are exogenous $\Longrightarrow$ GK suggest the magnitude of actual financial market shocks are depend on the leverage of FIs.

- FIs expect the CB to intermediate in states of the world in which their ICCs bind $\Longrightarrow$ an incentive for FIs to engage in adverse selection and/or moral hazard w/r/t the asset side of their balance sheets.

Gertler, Kiyotaki, and Queralto (JME, 2012): Introduction

- GKQ add equity issuance by FIs to the DSGE model of Gertler and Karadi (JME, 2011).

- FIs choose the source of and mix funds (*i.e.*, liabilities) employed to purchase assets.
  1. Risk on the FI balance sheets is an implicit choice variable.
  2. An FI can accept demand deposits, which are short-term liabilities, or issue equity, which is a long-term liability.
  3. $\Longrightarrow$ Equity imposes interest rate risk on FI balance sheets.

- GKQ argue moral hazard can be a problem in their DSGE model.
  1. FIs induce leverage on their balance sheets by funding assets by issuing more equity than accepting more demand deposits.
  2. Expectations of a central bank "bailout" is an incentive for FIs to leverage their balance sheets by issuing more equity.
  3. GKQ ask, "Can a CB engage in policies that present incentives to FIs that cause them to reduce the risk on their balance sheets by not engaging in moral hazard?"

GKQ: FI Balance Sheets

- Equity adds an additional margin to the FI's optimization problem.
- An FI funds assets on its balance sheets by selling equity or accepting demand deposits from households.
    1. The balance sheet constraint is $Q_t s_t = n_t + q_t e_t + d_t$, where q_t is the price of FI equity, e_t $\Longrightarrow$ the FI funds loans (*i.e.*, assets) with net worth (*i.e.*, inside equity), equity (sold to households), and deposits.
    2. FI net worth evolves as $n_t = r_{k,t} Q_{t-1} s_{t-1} - r_{e,t} q_{t-1} e_{t-1} - r_{d,t} d_{t-1}$, where $r_{e,t}$ is the return to equity $\Longrightarrow$ net worth is the return to loans net of the cost or equity and deposits.
- GKQ specify the fraction of assets a FI can divert for its own use, $\Theta(x_t)$, as the quadratic function $\Theta(x_t) = \theta\left(1 + \varepsilon x_t + \frac{\kappa}{2} x_t^2\right)$
    1. where $x_t = \dfrac{q_t e_t}{Q_t s_t}$ $\Longrightarrow$ of the ratio of the market value of equity to the market value of loan.
    2. $\Longrightarrow$ As x_t increases, the FI can abscond with more the value of the assets on its balance sheets.
    3. This induces a Tobin's q for $1/x_t$ $\Longrightarrow$ when equity is inexpensive compared with the market value of loans, this Tobin's q > 1.

Moral Hazard in GKQ's DSGE Model

- Moral hazard produces risk premiums in asset returns.
- Risk premiums are higher-order moments of the distributions of asset returns.
  1. Linearizing a DGSE model annihilates higher-order moments of state variables $\Longrightarrow$ eliminates risk premiums.
  2. GKQ use nonlinear methods to solve their NKDSGE model w/r/t a second-order approximation of FI balance sheet optimality and equilibrium conditions.
  3. The steady state is "risk-adjusted" $\Longrightarrow$ turn on the second moments of the state variables when a shock is realized.
- The risk-adjusted steady state aims to capture FI beliefs about chance the CB will issue debt when there is liquidity crisis.
  1. If FIs anticipate a low risk state of the world, their incentive is to increase leverage by issuing more equity.
  2. $\Longrightarrow$ If TFP is low, FI consume more by absconding with $\Theta(x_t)$ of their loans.
  3. When the CB is expected to lend to FIs in a crisis, FI leverage rises increasing the probability of a financial crisis.
- GKQ argue CBs lack technologies to commit credibly not to bailout illiquid FIs.
  1. $\Longrightarrow$ FIs are rational *ex ante* to expect a CB rescue because.
  2. CBs hold beliefs that the bailout raises *ex post* social welfare.

Farhi and Tirole (AER-PnP, 2009): Introduction

- FT study the strategic response of FIs when they (do not) anticipate a CB will (not) bailout failed projects $\Longrightarrow$ an externality among FIs.
- The externality is the return on a FI's assets depends on the leverage choices of other FIs.
- CB monetary policy is the source of the externality.
  1. Interest rate policy is not "targeted" $\Longrightarrow$ effects the aggregate economy (*i.e.*, all economic agents).
  2. However, some economic agents' welfare is more interest rate sensitivity than others $\Longrightarrow$ monetary policy alters real allocations. (*i.e.*, There are "winners" and "losers" when a CB changes its policy rate.)
- The more sensitivity is a FI to changes in the policy rate the greater the incentive to engage in adverse selection and/or moral hazard.
  1. FIs buy assets and issue liabilities that exhibit greater interest rate risk $\Longrightarrow$ more leveraged balance sheets.
  2. Greater leverage means the state of FI balance sheets react more to the state of the aggregate economy.
  3. Monetary policy becomes time-inconsistent $\Longrightarrow$ *Ex ante* CB commits not to lower the policy rate to bail out FIs, but *ex post* the CB does.
- FT argue CB bailout policies, such as the Greenspan and Bernanke puts, are more likely to payoff the more leveraged are FI balance sheets.

FT: The Set-Up of the Model, I

- FT's model is grounded in the problem that entrepreneurs cannot credible pledge returns from a project to outside investors studied by Holmström and Tirole (2011).
- The economy is real, it exists for three periods, $t = 0, 1, 2$, and consists of entrepreneurs and investors each taking addresses on distinct unit intervals.
- Investor utility is $\mathcal{V} = c_0 + u(c_1) + c_2$, where
  1. c_t is an investor's date t consumption and $u(\cdot)$ is increasing and concave.
  2. An investor has "relatively large" endowments e_0 and e_1 of the consumption good at dates $t = 0$ and 1.
- Entrepreneurs are risk neutral $\Longrightarrow$ utility is $\mathcal{U} = c_{E,0} + c_{E,1} + c_{E,2}$ and they own constant returns to scale (CRS) technologies, where
  1. $c_{E,t}$ is date t consumption of an entrepreneur and $\mathcal{A}$ is wealth of an entrepreneur at $t = 0$ (this is the entrepreneur's only endowment).
  2. Project scale $\mathcal{I}$ set by entrepreneurs at $t = 0 \Longrightarrow$ a fraction ρ_0 is pledged to investors.
  3. $\Longrightarrow$ Entrepreneurial rents on a successful project $= (\rho_1 - \rho_0)\mathcal{I}$ at date 2, where ρ_1 is gross project return at the end of date 1.
- Remember rent per unit of $\mathcal{I}$, $\rho_1 - \rho_0$, is a wedge created by two financial frictions.
  1. Entrepreneurial wealth is finite, $\mathcal{A} \in (0, \infty)$ and rents are not insurable.
  2. $\Longrightarrow$ Investors buy AD securities from entrepreneurs only up to $\rho_0\mathcal{I}$.
  3. Since rents lack a credible pledge, entrepreneurs become liquidity constrained in some states of the world.

FT: The Set-Up of the Model, II

- FT need a device to generate liquidity funding risk for entrepreneurs.
- Entrepreneurs choose between safe and risky technologies at $t = 0$.
- The safe technology costs $\mathcal{K}$ per unit of $\mathcal{I}_S$ more to operate compared with the cost $\mathcal{I}$ of the risky technology $\Longrightarrow \mathcal{K} > 1$.
- Liquidity shocks affect the risky technology at $t = 1$, but the safe technology is immune to these shocks.
  1. Risky projects are hit by liquidity shocks with probability $1 - \alpha$.
  2. Given a liquidity shock, an entrepreneur needs to replace the initial investment $\mathcal{I}$ one-for-one for the project to payoff at $t = 2$.
  3. An entrepreneur unable to raise the additional funds subsequent to a liquidity shock shuts down the project $\Longrightarrow$ payoff $= 0$.
  4. With probability α, a risky project suffers no liquidity shock and at $t = 2$ pays off $(\rho_1 - \rho_0)\mathcal{I}$.
- **Assumption 1**: When entrepreneurs expect no bail out from the CB, they engage only the safe technology if

$$\frac{1}{\alpha} > \mathcal{K} > 1 + (1 - \alpha)\rho_0.$$

$\Longrightarrow$ Necessary restrictions implying (*i*) the probability of no liquidity shock cannot be "too large" while (*ii*) the cost per unit of the safe technology has to be greater than the "expected return" to investors.

FT: The Set-Up of the Model, III

- FT give investors an outside opportunity to move their $t = 1$ endowment, e_1, to $t = 2$.
  1. Investors use a riskless asset to substitute e_1 intertemporally at $t = 1$.
  2. The riskless asset pays off one unit at $t = 2$ for every unit purchased.
  3. This gives investors a choice between investing in a risky project hit by a liquidity shock or the riskless asset at $t = 1$.

- All markets are perfectly competitive $\Longrightarrow$ agent actions are perfectly observable.

- However, several markets are closed by assumption at $t = 0$,
  1. no riskless asset market and no AD security markets to insure entrepreneurial rents against liquidity shocks.
  2. $\Longrightarrow$ an entrepreneur cannot hedge liquidity shock risk with investors or with other entrepreneurs.
  3. FT claim their results are robust to the open these markets.

- The CB aims to maximize $\mathcal{W} = \mathcal{V} + \beta\mathcal{U}$, where $\mathcal{W}$ is the economy's social welfare function and the weight on entrepreneurial utility is $\beta \in (0, 1)$.
  1. The policy rate $\mathcal{R}$, which is real, is the CB's tool for maximizing $\mathcal{W}$.
  2. Implicit is the CB's policy rate, $\mathcal{R} > 0$, does not necessarily dominate the date 1 riskless asset in rate of return.
  3. The "excess" return is rebated lump sum to investors $\Longrightarrow$ CB policy redistributes the consumption good to maximize $\mathcal{W}$.

FT: THE SET-UP OF THE MODEL, IV

- The CB faces the policy trade-off that dropping $\mathcal{R} \leq 1$
  1. provides an incentive for investors to invest in risky projects at date 0 and in failing projects at date 1, but
  2. is an incentive for investors to hold the date 1 riskless asset $\Longrightarrow$ its rate of return dominates $\mathcal{R}$.
- **Assumption 2**: Ignoring bailout costs, the CB maximizes $\mathcal{W}$ by lowering $\mathcal{R} \leq 1$, by restricting $\beta(\rho_1 - \rho_0) > 1 - \rho_0$.
  1. The restriction is equivalent to $(1 - \beta)\rho_0 > 1 - \beta\rho_1$.
  2. $\Longrightarrow$ The "annuity value" of a unit of pledged income to an investor has to be greater than
  3. the "discounted income lost" by not continuing a failing project that was hit by a liquidity shock.
- Redistributing scarce liquidity to entrepreneurs with failing projects
  1. is socially optimal for the CB whether it can commit to $\mathcal{R} = \rho_0$ or not
  2. under the restriction of assumption 2.

Commitment and Passive Monetary Policy, I

- The CB never permits $\mathcal{R} < \rho_0 \Longrightarrow$ a commitment to provide failing projects with liquid funds in place of the market (*i.e.*, investors).
- Define $y \equiv \Pr(\mathcal{R} = \rho_0)$ be the exogenous probability of a bailout by the CB, where $\mathcal{R} = \rho_0$ signals the state of the world in which the CB bails out failing projects.
- FT rule out the CB subsidizing failing projects with direct transfers $\Longrightarrow$ monetary (*i.e.* interest rate) policy creates incentives for investor to fund these projects.

Commitment and Passive Monetary Policy, II

- Entrepreneurs operating the safe technology set $\mathcal{I}_S^* = \dfrac{\mathcal{A}}{\mathcal{K} - \rho_0}$ because pledgeable returns are limited to $\rho_0 \mathcal{I}_S = \mathcal{K}\mathcal{I}_S - \mathcal{A}$.
- The safe technology yields the entrepreneur $\mathcal{U}_S = (\rho_1 - \rho_0)\mathcal{I}_S^*$, where $c_0 = 0$ and the gross return on the safe project is ρ_1.
- The risky technology presents entrepreneurs with a problem.
  1. If $\mathcal{R} > \rho_0$ at date 1, investors never supply liquidity to a failing project.
  2. $\Longrightarrow$ Investors have an outside opportunity in the riskless asset at $t = 1$.
  3. Only if $\mathcal{R} \leq \rho_0$, does a failing project obtain funds from investors to continue operating.
  4. Since $\mathcal{I} - \mathcal{A} = \alpha\rho_0\mathcal{I}$ is the entrepreneur's borrowing capacity when choosing the risky project, $\mathcal{I}^* = \dfrac{\mathcal{A}}{1 - \alpha\rho_0}$.
  5. $\Longrightarrow$ An entrepreneur running the risky project receives utility of $\mathcal{U} = (\rho_1 - \rho_0)[\alpha + (1 - \alpha)y]\mathcal{I}^*$, where $\alpha + (1 - \alpha)y$ is the probability the risky project continues to operate into date 2 (*i.e.*, either the project does not fail or investors provide liquidity to the failing project).

Commitment and Passive Monetary Policy, III

- Should entrepreneurs choose the riskless or risky project at $t = 0$?
- Which project gives an entrepreneur greater utility?
- *Iff* $\mathcal{U}_S \geq \mathcal{U}$, do entrepreneurs employ the safe technology.
  1. $\Longrightarrow \mathcal{I}_S^* \geq [\alpha + (1-\alpha)y]\mathcal{I}^*$, or $\mathcal{I}_S^* < \mathcal{I}^*$.
  2. Assumption 1 sets $\mathcal{K} > 1 + (1-\alpha)\rho_0 \Longrightarrow$ the safe technology is increasingly costly compared to the risky technology the larger is $\mathcal{K}$ relative to $1 + (1-\alpha)\rho_0$.
  3. $\Longrightarrow$ An incentive for entrepreneurs to increase the scale of risky projects, given $\mathcal{I}_S^*$.
- On the other hand, the probability a risky projects succeeds is less than the probability a safe project does, $\alpha + (1-\alpha)y < 1$.
  1. As y increases, the gap between these probability shrinks.
  2. Entrepreneurs see the risky project as a winner $\Longrightarrow$ greater likelihood of a CB bailout in case of bad liquidity shock.
- Under commitment, $\mathcal{I}_S^* \geq [\alpha + (1-\alpha)y]\mathcal{I}^*$ characterizes the equilibrium w/r/t the investment choices of entrepreneur.

Optimal Passive Monetary Policy under Commitment

- Suppose the CB adopts a monetary policy of "hands-off" or laissez-faire.
  1. The CB never bails out failing projects $\Longrightarrow \mathcal{Y} = 0$ and $\mathcal{R} = 1 > \rho_0$.
  2. Under this form of CB commitment, entrepreneurs never invest in the risky project $\Longrightarrow x = 0$, where x is the share of entrepreneurs operating the risky project.
  3. Or x is the fraction of the aggregate portfolio in the risky asset.
- The polar case is the CB always commitments to rescuing failing projects $\Longrightarrow \mathcal{Y} = 1,\ \mathcal{R} = \rho_0$, and $x = 1$.
- CB monetary policy is either laissez-faire or there are bailouts under commitment.
  1. Under laissez-faire, investor utility is $u(e_1)$ at $t = 1$, where $u'(e_1) = \mathcal{R} = 1$.
  2. Otherwise, at $t = 1$, $u(e_1 - \mathcal{I}_1)$ and $u'(e_1 - \mathcal{I}_1) = \mathcal{R} = \rho_0 < 1$, where investors supply $\mathcal{I}_1$ as aggregate liquidity to failing projects.
  3. Let investor utility be $\mathcal{V}(1) \equiv u(e_1)$ under laissez-faire or under bailouts it is $\mathcal{V}(\rho_0) \equiv u(e_1 - \mathcal{I}_1) - \mathcal{I}_1$.
- Optimal policy is decided by $\mathcal{V}(1) - \mathcal{V}(\rho_0) \gtreqless (\rho_1 - \rho_0)(\mathcal{I}^* - \mathcal{I}_S^*) - (1 - \rho_0)(1 - \alpha)\mathcal{I}^*$.
  1. *Iff* investor welfare under laissez-faire net of welfare under bailouts > benefit to entrepreneurs of the bailout is a policy of laissez-faire optimal.
  2. Under a bailout commitment, entrepreneurs benefit by scaling up projects, $(\rho_1 - \rho_0)(\mathcal{I}^* - \mathcal{I}_S^*)$, net of the costs of additional liquidity, $(1 - \rho_0)(1 - \alpha)\mathcal{I}^*$.
  3. The bailout commitment creates a wedge in $\mathcal{R}$ across the two CB policies that affects investor welfare and the entrepreneurs' net benefit of the risky project.

Monetary Policy without Commitment

- At date $t = 1$, the CB lacks a credible commitment to laissez-faire or bailouts.
- However, $\mathcal{R}$ still has to be fixed by the CB at one or ρ_0 during date 1.
  1. Since at $t = 0$ commitment to laissez-faire or bailouts is not credible for the CB, it expects entrepreneurs plan for bailouts.
  2. $\Longrightarrow$ The CB uses $\mathcal{I}_1$ as the economy's state variable, where $\mathcal{I}_1 \equiv (1 - \alpha)x\mathcal{I}^*$.
- Bailouts ($\mathcal{R} = \rho_0$) are optimal CB policy *ex post* if the welfare loss to investors of the interest wedge is less than the net benefits to entrepreneurs

$$\mathcal{V}(1) - \mathcal{V}(\rho_0) \leq \Big[\beta(\rho_1 - \rho_0) - (1 - \rho_0)\Big]\mathcal{I}_1.$$

  1. Investor welfare loss is a function of the "deep" primitives of the economy $\Longrightarrow$ investor decisions are passive given $\mathcal{R}$.
  2. Entrepreneurs choose the scale of the project conditional on $\mathcal{R} \Longrightarrow$ scale up $\mathcal{I}^*$ in the aggregate to generate income $\beta(\rho_1 - \rho_0)$ per unit net of the *ex post* per unit costs $(1 - \rho_0)$.
- Entrepreneurs know the chance of a CB bailout is higher the larger is $\mathcal{I}$.
  1. The inability of the CB to commitment credibly to $\mathcal{R}$ at $t = 0$ generates an externality.
  2. Gives entrepreneurs an incentive to increase their leverage.
  3. $\Longrightarrow$ By acting strategically, entrepreneurs compliment one another, which makes a CB bailout more likely *ex post*.

Equilibrium when the Central Bank Cannot Commit

- The laissez-faire equilibrium exists under no CB commitment.
  1. The CB finds it optimal to set $\mathcal{R} = 1$ conditional on entrepreneurs choosing the safe technology.
  2. $\Longrightarrow \mathcal{I}_1 = x = y = 0$ is an equilibrium.

- CB bailouts are an equilibrium, $x = y = 1$, if

$$\mathcal{V}(1) - \mathcal{V}(\rho_0) \leq \Big[\beta(\rho_1 - \rho_0) - (1 - \rho_0)\Big](1 - \alpha)\mathcal{I}^*,$$

  1. which is stricter constraint than is imposed by assumption 2.
  2. Entrepreneurs prefer the bailout equilibrium $\Longrightarrow$ Pareto dominates the laissez-faire equilibrium, where $x = y = 0$.
  3. Investors are no worse under the bailout equilibrium while entrepreneurs are better off $\Longrightarrow$ the latter could compensate the former making everyone better off.

- FT adopt the equilibrium selection device that the economy is in the bailout equilibrium, given multiple equilibria.

Risk-Taking, Leverage, and Interest Rate Policy

- The inequality condition $\mathcal{V}(1) - \mathcal{V}(\rho_0) \leq \Big[\beta(\rho_1 - \rho_0) - (1 - \rho_0)\Big]\mathcal{I}_1$ determines the sensitivity of entrepreneurial investment decisions to the state of the economy.
  1. Bailouts are more likely the larger is $\beta \Longrightarrow$ the CB gives entrepreneurial utility more weight in social welfare.
  2. Entrepreneurs add leverage the greater the probability of a liquidity shock $\Longrightarrow x \longrightarrow 1$ as $\alpha \longrightarrow 0$, under no commitment.
  3. This bad news creates entrepreneurial expectations the CB's put will payoff $\Longrightarrow$ entrepreneurs increase leverage as project returns become more convex.
  4. Even if α is small $\Longrightarrow 1 > \alpha\mathcal{K}$, $x = y = 0$ is an equilibrium under commitment to laissez-faire.
- If $(\rho_1 - \rho_0)\left(\mathcal{I}^* - \mathcal{I}_S^*\right) - (1 - \rho_0)(1 - \alpha)\mathcal{I}^* < \mathcal{V}(1) - \mathcal{V}(\rho_0) \leq \Big[\beta(\rho_1 - \rho_0) - (1 - \rho_0)\Big]\mathcal{I}_1$

  are satisfied, laissez-faire is optimal when the CB can commitment at $t = 0$.
- When commitment is impossible, CB policy is time-inconsistent
  1. The CB wants to claim its policy is $\mathcal{R} = 1$ at $t = 0 \Longrightarrow$ no bailouts because its offers greater welfare.
  2. Since this policy is not credible, $x \longrightarrow 1 \Longrightarrow$ entrepreneurs increase leverage.
  3. The CB is forced to renege on its no-bailout policy because bailing out failing projects becomes optimal *ex post*.
- FT argue the predictions of their model call for macro-prudential policy to regulate decisions a FI makes w/r/t the choice of assets on its balance sheet.

Introduction: Review

- The KM class of DSGE models center the transmission mechanism on changes in the expected value of durable goods .
- *Collateral Amplification*: Durable goods have value in facilitating exchange in credit markets besides the expected PDV of the income stream these stocks provide.
  1. Changes in the expected PDV of these durables goods alter the value of this collateral $\Longrightarrow$ alter the extent to which collateral constraints faced by borrowers bind.
  2. Given a contractionary shock to the expected PDV of the collateral falls, collateral constraints are tighter and if the shock propagates the drop in expected income streams, borrowers are forced to sell their durable goods to lenders below these valuations.
  3. $\Longrightarrow$ Durable goods are sold at fire sale prices because lenders place lower value on these durable goods than borrowers.

Introduction: Papers Covered in this section

- Goodfriend and McCallum (JME, 2007) develop a two sector monetary DSGE model $\Longrightarrow$ one sector produces capital and the other loans collateralized by capital and government debt.
    1. Capital and government debt have different valuations in collateralized trades involving credit $\Longrightarrow$ financial frictions is the collateral constraint and government debt and capital are imperfect substitutes.
    2. There are markets for deposits at FIs, short term government debt, and capital.
    3. Collateral constraints drive wedges between rates in the interbank market, government debt, capital, and the household SDF $\Longrightarrow$ which rate should the CB target to smooth shocks to inflation and stabilize real activity.

Introduction: Goodfriend and McCallum (JME 2007)

- GM's DSGE model contains a conventional financial accelerator, which they call a bank accelerator.
- There is also a "banking attenuator" in the DSGE model, which is an innovation according to GM.
- Expansionary monetary policy accelerates economic activity and simultaneously attenuates it.
  1. The BGG external finance premium (EFP) is reduced by a positive monetary policy shock.
  2. The price of capital, q_t, and its MP rises (conditional on NK nominal frictions), which increases capital's value in collaterlized trades $\Longrightarrow$ real activity is higher.
  3. However, households place more deposits with FIs in response to the monetary shock.
  4. The EFP $\uparrow$ $\Longrightarrow$ more liabilities on FI balance sheets.
- GM posit that FIs face costs when accepting deposits and producing loans $\Longrightarrow$ financial friction creates wedges between returns on disparate assets.

GOODFRIEND AND MCCALLUM (JME 2007): THE HOUSEHOLD'S PROBLEM

- The representative household's lifetime preferences are

$$\mathbf{E}_t\left\{\sum_{j=0}^{\infty}\beta^j\Big[\phi\ln C_{t+j} + (1-\phi)\ln(1-N_{S,t+j}-M_{S,t+j})\Big]\right\}, \quad \beta,\,\phi\in(0,\,1),$$

  where C_t is consumption and $N_{S,t}(M_{S,t})$ is labor supplied to the goods market (FIs).

- The budget constraint is

$$C_t + \frac{B_{t+1}}{P_{A,t}(1+R_{B,t})} + q_tK_{t+1} + \mathcal{T}_t + \frac{H_t}{P_{A,t}} + w_t(N_{D,t}+M_{D,t})$$

$$= w_t(N_{S,t}+M_{S,t}) + Y_{A,t}\left(\frac{P_t}{P_{A,t}}\right)^{1-\theta} + \frac{B_t}{P_{A,t}} + q_t(1-\delta)K_t + \frac{H_{t-1}}{P_{A,t}},\ \delta\in(0,\,1),$$

  where B_{t+1}, $P_{A,t}$, $R_{B,t}$, K_{t+1}, $\mathcal{T}_t$, H_t, w_t, $N_{D,t}$, $M_{D,t}$, P_t, and δ are the nominal government bonds the household takes from t into $t+1$, the aggregate price level, the nominal return on bonds, the capital stock the household takes from t into $t+1$, lump sum taxes, high powered money the household owns at the end of date t, the real wage, goods market labor demand, FI labor demand, the price of goods produced by the household, and the depreciation rate.

- Monopolistic competition lets the household choose P_t s.t. aggregate demand, $Y_{A,t}$ $\Longrightarrow Y_{A,t} = (P_t/P_{A,t})^{\theta}K_t^{\eta}(A_{1,t}N_t)^{1-\eta}$, where $A_{1,t}$ is a labor augmenting goods productivity shock and $\eta\in(0,\,1)$.

Goodfriend and McCallum (JME 2007): The FI's Problem

- The FI's balance sheet is $H_t = L_t - D_t$, where L_t is loans to households and household leave deposits, D_t, with the FI.
  1. The household's deposit decision is not explicit in its intertemporal choice problem because GM net the asset–liability positions of the household and FI.
  2. Free entry into a perfectly competitive FI market $\Longrightarrow$ zero profits for the FI.

- The demand for H_t operated through the liability side of the FI balance sheet because of the deposit in advance constraint $C_t = \mathcal{V}\dfrac{D_t}{P_{A,t}}$, where $\mathcal{V}$ is the velocity of H_t net of $L_t \Longrightarrow$ the FI is an extension of the household.

- The FI fixes its reserve ratio, $\mathcal{R}_R = 1 - \dfrac{L_t}{D_t} \Longrightarrow$ high powered money left with the CB.

- GM describe a loan management technology, which monitors or creates L_t using $M_{D,t}$, $q_t K_{t+1}$, and collateral, $b_{t+1} = B_{t+1}/[P_{A,t}(1 + R_{B,t})]$,

$$\frac{L_t}{P_{A,t}} = \left(b_{t+1} + \mathcal{K} A_{3,t} q_t K_{t+1}\right)^{\alpha} \left(A_{2,t} M_{D,t}\right)^{1-\alpha}, \quad \alpha \in (0, 1),$$

  where $\mathcal{K}$ is a scaling parameter to balance $A_{3,t} q_t K_{t+1}$ and b_{t+1}, $A_{3,t}$ is a "collateral monitoring" shock, and $A_{2,t}$ is the labor augmenting monitoring productivity shock.

- The loan monitoring function assumes that government bonds generate more loans per unit than does $q_t K_{t+1}$.

Goodfriend and McCallum (JME 2007): Intratemporal Optimality

- GM assume (*i*) the equilibrium is symmetric for household engaged in monopolistic competition and (*ii*) hold K_t constant at K for all t (except for deterministic growth).
- Let λ_t and ξ_t be the Lagrange multipliers on the household's budget and aggregate demand constraints $\Longrightarrow$ household intratemporal optimality conditions are

$$
\begin{aligned}
w_t \lambda_t &= \frac{1-\phi}{1-N_t-M_t}, \\
w_t &= \frac{1-\alpha}{M_t}\left(\frac{\phi}{C_t}-\lambda_t\right)\frac{C_t}{\lambda_t}, \\
w_t &= (1-\eta)A_{1,t}\left(\frac{\xi_t}{\lambda_t}\right)\left(\frac{K}{A_{1,t}N_t}\right)^{\eta}, \\
\frac{\xi_t}{\lambda_t} &= \frac{\theta-1}{\theta}, \\
\text{and} \qquad b_{t+1} &= \frac{B_{t+1}}{P_{A,t}\left(1+R_{B,t}\right)},
\end{aligned}
$$

for labor supply, labor demand for the FI, labor demand in the goods market, the monopolistic competition restriction across household and goods market resources, and the real value of collateral.

Goodfriend and McCallum (JME 2007): Intertemporal Optimality

- The household intertemporal optimality conditions are

$$q_t = \left(\frac{\phi}{C_t} - \lambda_t\right)\frac{\mathcal{K} q_t \Omega_t}{\lambda_t} + \beta(1-\delta)\mathbf{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t} q_{t+1}\right\} + \eta\beta\mathbf{E}_t\left\{\frac{\xi_{t+1}}{\lambda_t}\left(\frac{A_{1,t+1}N_{t+1}}{K}\right)^{1-\eta}\right\},$$

$$1 = \left(\frac{\phi}{C_t} - \lambda_t\right)\frac{\Omega_t}{\lambda_t} + \beta\left(1 + R_{B,t}\right)\mathbf{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\frac{P_t}{P_{t+1}}\right\},$$

for capital and nominal government bonds, where $\Omega_t \equiv \dfrac{\alpha C_t}{b_{t+1} + \mathcal{K} A_{3,t} q_t K_{t+1}}$ is the real value of the collateral services provided by the real market value of discounted government bonds and of capital in generating loans.

Goodfriend and McCallum (JME 2007): Interest Rate Wedges, I

- Arbitrage demands equality across returns in expectations weighted by risk.
- Define a riskless asset that provides a benchmark to measure interest rate wedges $\Longrightarrow$ a unit discount bond (in zero net supply) that pays $\dfrac{1}{1+R_{T,t}} = \mathbf{E}_t\left\{\dfrac{\lambda_{t+1}}{\lambda_t}\dfrac{P_t}{P_{t+1}}\right\}$.
- The nominal government bond Euler equation becomes

$$\frac{1+R_{B,t}}{1+R_{T,t}} = 1 + \left(\lambda_t - \frac{\phi}{C_t}\right)\frac{\Omega_t}{\lambda_t}.$$

- The nominal bond rate wedge equals the value of collateral services measured by the gap between the shadow price of an extra unit of output and the MU of consumption.
- The interbank market rate is $R_{IB,t} \Longrightarrow$ rate at which FIs trade excess reserves, $\mathcal{R}_R$.
  1. One arbitrage is that the FI could borrow in the interbank market and loan to the household.
  2. A FI buys or sells funds to equate MC and MP of loan production/monitoring $\Longrightarrow$ assume the FI incurs the costs to conduct this arbitrage.
  3. $\Longrightarrow$ real MC = the factor price, w_t, divided by its MP, $\dfrac{\partial L_t/P_{A,t}}{\partial M_{D,t}} = \dfrac{1-\alpha}{M_t}\dfrac{L_t}{P_{A,t}}$ $\Longrightarrow \dfrac{\mathcal{V}w_tM_t}{(1-\alpha)(1-\mathcal{R}_R)C_t}$, where $\dfrac{L_t}{P_{A,t}} = \dfrac{(1-\mathcal{R}_R)C_t}{\mathcal{V}}$.
  4. Arbitrage a unit of interbank funds $\Longrightarrow \dfrac{1+R_{T,t}}{1+R_{IB,t}} = \left[1 + \dfrac{\mathcal{V}w_tM_t}{(1-\alpha)(1-\mathcal{R}_R)C_t}\right]$.

GOODFRIEND AND MCCALLUM (JME 2007): INTEREST RATE WEDGES, II

- However, households do cover the cost of non-collaterlized loan production and monitoring $\Longrightarrow$ the external finance premium = $\dfrac{\mathcal{V} w_t M_t}{(1-\alpha)(1-\mathcal{R}_R)C_t}$.
- Similar arbitrage arguments are employed to show

$$\frac{1 + R_{L,t}}{1 + R_{IB,t}} = \left[1 + \frac{\mathcal{V} w_t M_t}{(1-\mathcal{R}_R)C_t} \right],$$

  where $R_{L,t}$ is the rate on collateralized loans and $1-\alpha$ is eliminated because this credit contract has no need to be monitored $\Longrightarrow$ these borrowers earn a rent or are compensated for tying up their assets to secure credit.
- Since the external finance premium of collaterlized loans = $\dfrac{\mathcal{V} w_t M_t}{(1-\mathcal{R}_R)C_t}$, there is a wedge driven between rates on non-collaterlized and collaterlized loans $\Longrightarrow R_{L,t} - R_{IB,t} = (1-\alpha)(R_{T,t} - R_{IB,t})$.
- The wedge between the return on deposits, $R_{D,t}$ and $R_{IB,t}$ is the cash the FI leaves at the CB, which accrues no interest $\Longrightarrow \dfrac{R_{D,t}}{R_{IB,t}} = 1 - \mathcal{R}_R$.
- This vector of rates is not often found in new Keynesian DSGE models that rely on Woodfordian natural rate arguments.
- Instead, a CB aiming for price (or inflation) stability has to consider which private short asset to trade for its liability, H_t, when setting its intermediate rate target.